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Abstracts

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A Conformal Mapping Between n-Surfaces with Orthogonal Tangent Planes in E^{2n}

Let $M, \bar{M}$ be n-dimensional surfaces in Euclidean E^{2n} with orthogonal tangent planes and the mapping $f : M \to \bar{M}$. There is the mapping $\Omega : TM \to T^{\perp}M$, where $\Omega X = df X, X \in TM$.

There are metrics $g(X,Y) =< X, Y >$, $\bar{g}(X,Y) =< df X, df Y >$. Consider ∇ to be the Levi-Civita connection of g, $\bar{\nabla}$ to be the Levi-Civita connection of $\bar{g}$. We denote by $R, \bar{R}, R^{\perp}, \bar{R}^{\perp}$ the curvature tensor associated with $\nabla, \bar{\nabla}, \nabla^{\perp}, \bar{\nabla}^{\perp}$, respectively.

Then the Gauss and Weingarten formulas are given, respectively, by

$$\partial_X Y = \nabla_X Y + \alpha(X,Y), \quad \partial_X \xi = -A_\xi X + \nabla^{\perp}_X \xi,$$

where $X, Y \in T^1_0(M)$, ∇ is a Levi-Civita connection of $g(X,Y) = < X, Y >$, α is the second fundamental form, $\nabla^{\perp}$is a normal connection, $A_\xi \in T^1_1(M)$, $\xi \in TM^{\perp}$, $< \alpha(X,Y), \xi >= g(A_\xi X, Y)$.

The equation of Gauss and Codazzi are given by

$$R(X,Y,Z,U) = g(R(X,Y)Z,U)$$

$$=< \alpha(Y,Z), \alpha(X,U > - < \alpha(X,Z), \alpha(Y,U > .$$

We have

$$\bar{\nabla}_X Y = \Omega^{-1} \nabla^{\perp}_X \Omega Y, \quad \bar{R}(X,Y)Z = \Omega^{-1} R^{\perp}(X,Y) \Omega Z.$$

Let $\bar{g}, \bar{\alpha}$ be the first and the second fundamental forms of $\bar{M}$, respectively. We have $\bar{g}(X,Y) =< df X, df Y >=< \Omega X, \Omega Y >$,

$$(\partial_X df(Y))_p - (df(\bar{\nabla}_X Y)_p = \bar{\alpha}(X,Y)_p, \quad \bar{\alpha}(X,Y) = -A_{\Omega X} Y,$$

$$< \bar{A}_X \Omega Y, \Omega Z >=< X, \bar{\alpha}(Y,Z) >, \quad \bar{A}_X \Omega Y = -\alpha(X,Y),$$

$f : M \to \bar{M}$ is the conformal mapping. We have $\bar{g} = k^2 g$,

$$\alpha(X,Y) = -\frac{1}{k^2}\Omega\bar{\alpha}(X,Y), \quad R(X,Y,Z,U) = \frac{1}{k^2}\bar{R}(X,Y,Z,U).$$

$$g(\bar{R}^{\perp}(\Omega Z,\Omega U)X,Y) = \frac{1}{k^2}\bar{g}(\Omega^{-1}R^{\perp}(Z,U)\Omega X,Y).$$

Theorem 1. *Let $M,\bar{M}$ be n-dimensional surfaces in Euclidean E^{2n} with orthogonal tangent planes. Let $f : M \to \bar{M}$ be the conformal mapping. Then $R,\bar{R}$ the curvature tensors associated with $\nabla,\bar{\nabla}$ are proportional.*

Theorem 2. *Let $M,\bar{M}$ be n-dimensional surfaces in Euclidean E^{2n} with orthogonal tangent planes. Let $f : M \to \bar{M}$ be the conformal mapping. Then $R^{\perp},\bar{R}^{\perp}$ the curvature tensors associated with $\nabla^{\perp},\bar{\nabla}^{\perp}$ must satisfy this equation*

$$\bar{R}^{\perp}(\Omega X,\Omega Y)Z = \Omega^{-1}R^{\perp}(X,Y)\Omega Z.$$