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Abstracts

GOLUBYATNIKOV, VLADIMIR P., *Sobolev Institute of Mathematics, Novosibirsk, Russia*; *email*: `glbtn@math.nsc.ru`

Reconstruction of Compact Convex Bodies in the Complex Euclidean Spaces from Their Projections

We consider a variation of a classical problem: if projections of two convex compact bodies in a complex Euclidean space on any plane are congruent each other with respect to some group of linear transformations, how different can be these convex bodies?

In the case of the real Euclidean spaces the first results in this direction were obtained by W. Süss [2].

Since accounting of the wave effects in the problems of reconstruction of the multidimensional objects from their projections is often connected with the Fourier transformations, the complexification of the geometrical optics problem seems to be quite natural.

We shall denote by $V(\omega)$ the orthogonal projections of the body $V \subset \mathbb{C}^n$ on the subspace orthogonal to a unit vector $\omega \in S^{2n-1} \subset \mathbb{C}^n$.

Lemma. *Let $V_1, V_2 \subset \mathbb{C}^n$ be compact convex bodies in $\mathbb{C}^n$, and $\Omega_2 \subset S^{2n-1}$ is a subset, which intersects all the big circles on the sphere S^{2n-1} and contains a basis of $\mathbb{C}^n$.*

If the projections $V_1(\omega)$, $V_2(\omega)$ are homothetic each other for all $\omega \in \Omega_2$, (the homothety coefficient is not assumed to be constant, independent on the direction of projection ω), and their projections on any $(n-2)$-dimensional subspace have no $U(1)$-symmetries, then V_1 and V_2 can be superposed in $\mathbb{C}^n$ by a homothety with a complex coefficient.

Theorem. *Let V_1, V_2 be convex compact bodies in $\mathbb{C}^n$, $n > 2$, $1 < k < n$, and their projections on any k-dimensional complex plane are congruent each other with respect to action of the group $U(k)$ and have no $U(k)$-symmetries.*

If projections of any of these bodies on different $(k-1)$-dimensional complex planes are pairwise non-congruent with respect to the group $U(k-1)$, and projections of any of these bodies on different complex lines are pairwise non-congruent with respect to the group $U(1)$, then the bodies V_1, V_2 can be superposed in $\mathbb{C}^n$ by composition of a parallel *translation and a homothety with a complex homothety coefficient λ, such that $|\lambda| = 1$.*

Analogous result can be obtained for $(n-k)$-convex and $(n-k)$-visible compact bodies in $\mathbb{C}^n$. In the complex analysis the complements to the $(n-k)$-convex bodies in $\mathbb{C}^n$ are called $(k-1)$-linearly concave domains and are often considered in the problems connected with the Integral Geometry, see [1].

From the general mathematical viewpoint it would be interesting to examine similar problems in the vector spaces with other additional structures, such as the spaces with the quaternions or the dual numbers action etc.

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References

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[2] W. Süss. Zusammensetzung von Eikörpen und homotetische Eiflächen, *Tôhoku Math. J.*, **35**:47–50, 1932.