International Conference on Analysis and Geometry
August 30 – September 3, 1999, Novosibirsk, Russia

Abstracts

HEDBERG, LARS INGE, *Linköping University, Sweden*;
email: `lahed@mai.liu.se`

Nonlinear Potential Theory and Stability

We consider a stability problem for an open set $\Omega \subset \mathbb{R}^n$, which is motivated by work of P. Lindqvist on the dependence on p of the first eigenvalue of the p-Laplacian in Ω. This eigenvalue is by definition the least real λ such that the equation

$$\operatorname{div}(|\nabla u|^{p-2}\nabla u) + \lambda |u|^{p-2}u = 0$$

has a nontrivial solution $u \in W_0^{1,p}(\Omega)$. Here $1 < p < \infty$, and $W_0^{1,p}(\Omega)$ is the closure of $C_0^\infty(\Omega)$, the compactly supported C^∞-functions on Ω, in the $W^{1,p}$-norm $\|u\|_{1,p}$, defined by

$$\|u\|_{1,p}^p = \int_\Omega (|u|^p + |\nabla u|^p)\, dx.$$

Our stability problem is the following: *Let $1 < q < p$. For which bounded open $\Omega \subset \mathbb{R}^n$ is it true that*

$$W_0^{1,p}(\Omega) = W^{1,p}(\Omega) \cap W_0^{1,q}(\Omega)?$$

In a recent joint paper with Tero Kilpeläinen [2] a number of equivalent necessary and sufficient conditions for Ω to have this stability property were given. As a background for this result we review some of the necessary nonlinear potential theory, including capacities, thin sets, the Kellogg property, and fine continuity (see e.g. [1]).

References

[1] D. R. Adams and L. I. Hedberg. *Function Spaces and Potential Theory.* Springer-Verlag, Berlin Heidelberg, 1996.

[2] L. I. Hedberg and T. Kilpeläinen. On the stability of Sobolev spaces with zero boundary values. *Math. Scand.*, (to appear).