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Abstracts

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Gradients Without Rank-One Connections

We consider the following question. Given a finite set $\mathcal{A} \subset \mathbb{R}^{n\times m}$ of matrices, does there exist a nonaffine lipschitz map $f : \Omega \to \mathbb{R}^n$ on a domain $\Omega \subset \mathbb{R}^m$ such that

$$\nabla f(x) \in \mathcal{A} \text{ a.e. in } \Omega?$$

Of course such an f can be constructed provided there are $A, B \in \mathcal{A}$ such that $\operatorname{rank}(A - B) = 1$. This led to the question whether the existence of such a rank-one connection is also a necessary condition. From work by Šverák (see [2]) it was known that for $\operatorname{card}(\mathcal{A}) \leq 3$ this is true, even if one considers "approximative solutions" (e.g. in the sense of Young measures) of the differential inclusion. Whereas these approximative solutions exist in the case of four matrices (see e.g. [1]), we proved in joint work with M. Chlebík that no such exact solutions can be found. On the other hand, together with D. Preiss we gave an example of a nonaffine map using five symmetric gradients without rank-one connections. Nonexistence results are related to the regularity properties of maps of bounded distortion, whereas existence is linked to a geometric approach to rank-one convexity.

References

[1] L. Tartar. Some Remarks on Separately Convex Functions. In: *Microstructure and Phase Transition.* IMA Volumes in Mathematics and its Applications, **54**:192–204, 1994.

[2] V. Šverák. New examples of quasiconvex functions. *Arch. Rational Mech. Anal*, **119**(4):293–300, 1992.