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Abstracts

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Two-Generator Groups with Real Parameters

The well known Thurston theorem says that there are eight geometries on 3-manifolds. In particular, it was shown that in a sense most of 3-manifolds can be equipped with the complete metric of the constant negative curvature; i. e. they are hyperbolic manifolds. The classification problem for hyperbolic 3-manifolds and orbifolds is equivalent to the classification problem for discrete subgroups of $PSL(2,\mathbb{C})$.

It is quite reasonable to distinguish the class of two-generator discrete subgroups of $PSL(2,\mathbb{C})$. In particular, these groups appear in various extremal problems and happen to be the groups of the hyperbolic manifold and orbifold of minimal volume.

The necessary and sufficient conditions for discreteness of two-generator groups with invariant plane were previously known (Rosenberger, Matelski, Gilman and Maskit in oriented case; Klimenko and Sakuma in non-oriented). As for subgroups of $PSL(2,\mathbb{C})$ without invariant plane the known results provide either some necessary or some sufficient conditions.

In 1998 Gehring, Gilman, and Martin [1] suggested to study the class of two-generator groups $\langle f,g\rangle$ with real parameters $\beta = \mathrm{tr}^2 f - 4$, $\beta' = \mathrm{tr}^2 g - 4$ and $\gamma = \mathrm{tr}[f,g] - 2$. They gave the full description of discrete groups from this class in the case $\beta' = -4$, i. e. of a subclass of groups $\langle f,g\rangle$ with g being a half-turn. It can be seen that these groups have an invariant plane.

The first author has studied the class $\mathcal{G}$ of two-generator non-elementary and non-Fuchsian groups with real $\mathrm{tr}\, f$, $\mathrm{tr}\, g$, and $\mathrm{tr}\,[f,g]$ (see [2] and references therein).

In the present work we complete the classification of the discrete groups from $\mathcal{G}$. This means that we are able to obtain the full description of two-generator groups with real parameters β, β', and γ, since they are discrete extensions of groups from $\mathcal{G}$.

The method of the proof is geometric and gives rise to interesting hyperbolic orbifolds.

References

[1] F. Gehring, J. Gilman, G. Martin. *Discrete groups with real parameters.* Preprint, 1998.

[2] E. Klimenko. *A special class of 2-generator Kleinian groups.* Preprint, 1994.