International Conference on Analysis and Geometry

August 30 – September 3, 1999, Novosibirsk, Russia

Abstracts

KOROBKOV, MIKHAIL V., *Sobolev Institute of Mathematics, Novosibirsk, Russia*; *email*: `strizh@nsu.ru`

On Stability of Classes of Lipschitz Mappings Generated by Compact Sets of Linear Mappings

Let LIP denote the space of mappings $g : \Delta \to \mathbb{R}^m$, defined on domains in $\mathbb{R}^n$ and locally satisfying a Lipschitz condition. A class $\mathfrak{G}$ of mappings $g \in LIP$ is said to be ω-stable (see [1]), if it satisfies some normality conditions from [1] and there exists a nonnegative real function $\sigma = \sigma(\varepsilon)$, $\varepsilon \geq 0$, such that

1) $\sigma(\varepsilon) \to \sigma(0) = 0$ as $\varepsilon \to 0$;

2) for each map $f : \Delta \to \mathbb{R}^m$ $\;\omega(f, \mathfrak{G}) \leq \sigma(\Omega(f, \mathfrak{G}))$.

Here

$$\omega(f, \mathfrak{G}) = \sup_{B \subset \mathbb{R}^n} \omega_B(f, \mathfrak{G})$$

and

$$\Omega(f, \mathfrak{G}) = \sup_{x \in \Delta} \{\lim_{r \to 0} \sup \omega_{B(x,r)}(f, \mathfrak{G})\},$$

where in their turn $B(x, r)$ is a ball in $\mathbb{R}^n$ and

$$\omega_B(f, \mathfrak{G}) = \inf_{g : B \to \mathbb{R}^m, g \in \mathfrak{G}} \{r^{-1} \sup_{y \in B} |f(y) - g(y)|\}.$$

Definition. A set $U \subset M^{n \times m}$ is said to be q-connected, if there doesn't exist a family of sets $\{U_\alpha\}_{\alpha \in A}$ so that $U = \cup_{\alpha \in A} U_\alpha$ and for any $\alpha \in A$

(i) $U_\alpha \neq U$;

(ii) $U_\alpha \cap cl(U \setminus U_\alpha) = \emptyset$;

(iii) for any $x \in U_\alpha$ if $\alpha \neq \beta \in A$ then there exists a quasiconvex function $h : M^{n \times m} \to \mathbb{R}$ such that $h(x) > \sup_{y \in U_\beta} h(y)$.

Let $SCC_{qc}(G)$ denote the set of q-connected components of G. Determine the class

$$\mathfrak{Z}^{qc}(G) = \left\{ g \in LIP \;\middle|\; \begin{array}{l} \text{there exists } K \in SCC_{qc}(G) \\ \text{such that } g'(x) \in K \text{ a.e.} \end{array} \right\}.$$

Theorem 1. *Suppose G is a nonempty compact set in $M^{n\times m}$. If all the sets $K \in SCC_{qc}(G)$ are quasiconvex, then the class $\mathfrak{Z}^{qc}(G)$ is ω-stable.*

The proof of Theorem 1 relies on the following two results.

Theorem 2. *A nonempty compact set $G \subset M^{n\times m}$ satisfies the condition of Theorem 1 if and only if there exists a family of finite collections of quasiconvex compact sets G_i^α so that*

$$G = \bigcap_{\alpha\in A} \bigcup_{i=1}^{k_\alpha} G_i^\alpha,$$

where $G_i^\alpha \cap G \cap G_j^\alpha = \emptyset$ if $\alpha \in A$ and $i \neq j$.

Theorem 3. *Suppose $\mathfrak{G}$ is a nonempty class of mappings in LIP and there exists a family of finite collections of ω-stable classes $\mathfrak{G}_i^\alpha$ so that*

$$\mathfrak{G} = \bigcap_{\alpha\in A} \bigcup_{i=1}^{k_\alpha} \mathfrak{G}_i^\alpha,$$

where $\mathfrak{G}_i^\alpha \cap \mathfrak{G} \cap \mathfrak{G}_j^\alpha = \emptyset$ if $\alpha \in A$ and $i \neq j$. Then the class $\mathfrak{G}$ is ω-stable.

The following result illustrates the meaning of the notion of q-connectedness.

Theorem 4. *Suppose $f \in LIP$ is a differentiable map. Then* $\operatorname{Im} f'$ *is a q-connected set in $M^{n\times m}$.*

The research was supported by the Federal Program "Leading Scientific Schools" (Grant 96–15–96291), by the INTAS (Grant 97–0170), and by the RFBR (Grant 99–01–00517).

References

[1] A. P. Kopylov. On stability of isometric mappings. *Sibirsk. Mat. Zh.*, **25**(2):132–144, 1984.

[2] M. V. Korobkov. On some generalization of the notion of connectedness and its application to differential calculus and the theory of stability of classes of mappings. *Dokl. Akad. Nauk*, **363**(5):590–593, 1998.