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Abstracts

MARTIO, OLLI, *University of Helsinki, Finland*;
email: martio@cc.helsinki.fi
RYAZANOV, VLADIMIR, *Institute of Applied Mathematics and Mechanics, Donetsk, Ukraine*; *email*: ryazanov@iamm.ac.donetsk.ua

The Chernavskii Theorem and Quasiregular Mappings

The well-known Stoilow (1938) theorem of the topological function theory states that discrete open mappings on the plane or, more generally, on Rimannian surfaces are compositions of analytic functions and homeomorphisms.Thus, with the topological point of view, in the 2-dimensional case discrete open mappings form the same mapping class as analytic functions.

In the space quasiregular mappings give a natural generalization of the geometric and topological aspects of the theory of analytic functions. The Liouville theorem due to Reshetnyak (1967) says that a non-constant 1-quasiregular mapping $f : D \to \mathbf{R}^n$, $n \geq 3$, in the domain $D \subset \mathbf{R}^n$ is the restriction of a Möbius transformation of $\overline{\mathbf{R}^n}$ to D and thus a homeomorphism. Contrary to the plane case Q-quasiregular mappings in the space may have branching only if the distortion coefficient Q is large enough. By another theorem of Reshetnyak (1967) non-constant quasiregular mappings are discrete and open.

By the Chernavskii theorem (1964) topological dimension of the branch set for discrete open mappings between n-manifolds is at most $n-2$. Furthermore, Chernavskii conjectured that the dimension is equal to $n-2$. Church and Timourian (1978) gave counterexamples to the Chernavskii conjecture but their examples are not quasiregular mappings.

We develope the theory of the embedding dimension $\dim_E$ from compacta in $\mathbf{R}^n$ to closed sets in manifolds and show for discrete open mappings $f\colon M \to N$ of n-manifolds, $n \geq 2$, that either the branch set B_f is empty or $\dim_E f(B_f) \geq n-2$ and, furthermore, that $\dim_E f(B_f) = n-2$ with the only exception $n = 3$. Moreover, with the same exception, $\dim_E B_f \leq n-2$ and either $\dim_E B_f = n-2$ or $M \setminus B_f$ is simply connected in charts of M. In particular, the results hold for quasiregular mappings in $\mathbf{R}^n$. For PL open mappings the embedding dimension coincides with the topological dimension, i.e., the Chernavskii conjecture holds for such mappings.