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Abstracts

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On Hyperelliptic Involutions of Riemann Surfaces and 3-Manifolds

A 3-dimensional manifold M^3 is said to be *hyperelliptic* if there is an involution τ such that the quotient space $M^3/\langle\tau\rangle$ is homeomorphic to the 3-dimensional sphere. A hyperbolic 3-manifold M^3 can be defined as the quotient space $M^3 = H^3/G$, where G is a discrete group of isometries of the 3-dimensional hyperbolic space H^3, acting without fixed points. Hyperelliptic 3-manifolds can be considered as natural generalizations of hyperelliptic Riemann surfaces.

The following interesting problem was formulated by J. Birman [1]. Let K be a knot in the 3-sphere and let $\mathcal{M}(K)$ be its two-fold branched covering. We say that two knots K and L are *equivalent* if and only if corresponding manifolds $\mathcal{M}(K)$ and $\mathcal{M}(L)$ are homeomorphic. The problem is to describe classes of equivalent knots. It was shown in [2] that the number of inequivalent hyperelliptic involutions of any closed oriented hyperbolic 3-manifold M^3 is bounded by a constant C not depending on M^3, and the best known bound is $C = 9$. Recall the classical fact that a hyperelliptic involution of a hyperbolic Riemann surface is unique.

The small volume closed oriented hyperbolic 3-manifolds were listed by A. Fomenko and S. Matveev and independently by C. Hodgson and J. Weeks. It was shown in [3] that these manifolds can be obtained by Dehn surgeries on components of the two-component link, known as the *Whitehead link*. Denote by $\mathcal{W}(m/n; p/q)$ the 3-manifold obtained by Dehn surgeries on the Whitehead link $\mathcal{W}$ with surgery parameters m/n and p/q corresponding to its components.

Theorem 1. *Let M be one of the following 3-manifolds:*
1) $M = \mathcal{W}(1/n; 1/q)$, *where n and q are integers, $n \neq q$;*
2) $M = \mathcal{W}(1/n; 2/q)$, *where n is integer and q is odd, $n \neq q$;*

3) *$M = \mathcal{W}(2/n; 2/q)$, where n and q are odd, $n \neq q$.*
Then the manifold M has three inequivalent hyperelliptic involutions.

Moreover, corresponding branching sets are described in [4].

Theorem 2. *Let $M = \mathcal{W}(1/n; 1/n)$ be a 3-manifold as above. Then there is the dihedral group of order 8 of homeomorphisms of the manifold M which contains three inequivalent hyperelliptic involutions. Moreover, corresponding branching sets are described in* [4].

The proofs of theorems are constructive and use methods of low-dimensional topology and knot theory.

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References

[1] R. Kirby. Problems in Low-Dimensional Topology. In: *Geometric Topology, 1993 Georgia International Topology Conference.* (W. H. Kazez, editor). AMS/IP Studies in Advanced Mathematics. **2**(2).

[2] M. Reni, B. Zimmermann. *On Hyperelliptic involutions of hyperbolic 3-manifolds.* Preprint Series, Department of Mathematics, Trieste University, 1998.

[3] A. Mednykh, A. Vesnin. Covering Properties of small volume hyperbolic 3-manifolds. *Journal of Knot Theory and Ramifications*, **7**(3):381–392, 1998.

[4] A. Mednykh, A. Vesnin. *Hyperelliptic involutions of small volume hyperbolic 3-manifolds.* Preprint Series, Department of Mathematics, University of Helsinki, No. **189**, 1998 (to appear in Sib. Math. J.).