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Abstracts

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Weak and Classical Solutions for the Hele–Shaw Problem

The first topic of this talk is a method of constructing solutions to well-posed Hele-Shaw problems with general boundary conditions on the given boundary. Under minimal restrictions on the boundary and initial data, this method permits us to prove the existence of a weak solution. It also yields a comparison theorem for solutions, a theorem about the continuity of a solution with respect to boundary and initial data, and some estimates for BV–solutions.

Another topic is the study of global-in-time classical solutions to well-posed free boundary problems for the Hele-Shaw flow in two dimensions.

Earlier, weak solutions to well-posed Hele-Shaw problems were studied using variational inequalities by many authors (see, e.g., [1,2,3]; for details, consult [4,5]).

Locally in time, the classical solvability of the Hele–Shaw problem was established in [6] for analytic solutions in strip-like domains with analytic initial data and later in [7] for the Laplace equation with time-independent given data in arbitrary domains.

The principal difficulties in the study of global solvability originate in the very nature of the problem. In a solution to the Hele–Shaw problem, the pressure of the liquid in the unknown domain depends on time as a parameter — yet, the first energy estimates do not guarantee its smoothness with respect to time. Local smoothness in spatial variables may be lost near the free boundary. Because of these difficulties, the global-in-time classical solvability is proved only for domains with special geometry (strip-like or star-shaped ones) and only under the Dirichlet condition on the known boundary (where the given pressure depends only on time).

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