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Abstracts

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Pure Scales of Banach Spaces

The real method spaces $\overline{X}_{\theta,\overline{p}}$ with a vector-valued parameter $\overline{p} \in \mathbf{R}_n^+$ were introduced and studied by E. Nursultanov in [1]. The spaces $\overline{X}_{\theta,\overline{p}}$ may be defined inductively as follows

$$\overline{X}_{\theta,p_1,\ldots,p_{n-1},p_n,p_{n+1}} = (\overline{X}_{\theta,p_1,\ldots,p_{n-1},1}, \overline{X}_{\theta,p_1,\ldots,p_{n-1},\infty})_{\alpha,p_{n+1}},$$

where $1-\alpha = 1/p_n$ and $\overline{X}_{\theta,p}$ is the standard Lions–Peetre construction.

The main result of the present work is an introduction of the corresponding spaces for infinite sequences $\{p_n\}_{n\in\mathbf{N}}$ with $1 < p_n < \infty$ as the second parameter.

If we define the spaces $\overline{X}_{\theta,p}$ in a more delicate manner, i.e., if we use the norms

$$\|x\|_{\theta,p}^p = p\theta(1-\theta)\int_0^\infty \left(\frac{K(t,x)}{t^\theta}\right)^p \frac{dt}{t}, \qquad \|x\|_{\theta,\infty} = \sup_t \frac{K(t,x)}{t^\theta},$$

then the embeddings

$$\overline{X}_{\theta,p_1,\ldots,p_{n-1},1} \subset \overline{X}_{\theta,p_1,\ldots,p_{n-1},p_n,1} \subset \overline{X}_{\theta,p_1,\ldots,p_{n-1},p_n,\infty} \subseteqq \overline{X}_{\theta,p_1,\ldots,p_{n-1},\infty}$$

with the *unit* constant easily follows from the definition.

It turns out that limiting spaces for these monotone sequences of spaces exist and coincide. This limiting space is denoted by $\overline{X}_{\theta,\mathbf{p}}$ where $\mathbf{p} = \{p_n\}_{n\in\mathbf{N}}$, and the family of these spaces is called *a pure scale.*

Simple properties of these spaces were studied:

1. $\overline{X}_{\theta,\mathbf{p}}$ is a real method space,

2. $X_0 \cap X_1$ is dense in $\overline{X}_{\theta,\mathbf{p}}$,

3. $\|x\|_{\theta,\mathbf{p}} = \lim_{n\to\infty} \|x\|_{\theta,\overline{\mathbf{p}}_n}$, where $x \in X_0 \cap X_1$ and $\|x\|_{\overline{\mathbf{p}_n}}$ is the norm in Nursultanov spaces, corresponding to the finite sequence $\overline{\mathbf{p}}_n = (p_1, \dots, p_n)$,

4. If $X_0 \cap X_1$ is dense in X_0 and in X_1, then $\overline{X}^*_{\theta,\mathbf{p}} = \overline{X^*}_{\theta,\mathbf{p}'}$ where $1/p'_n = 1 - 1/p_n$.

The main result on coincidence of the limiting spaces is based on

Lemma. *If norms of the spaces in a Banach couple $\{X_0, X_1\}$ are equivalent and $\|x\|_1 \le \|x\|_0 \le c\|x\|_1$, then*

$$\|x\|_{\theta,\infty} \le \|x\|_{\theta,1} \le (1 + \theta(1-\theta)\log c)\|x\|_{\theta,\infty}.$$

We suppose that corresponding Lorentz spaces of new type may applied to studying of geometry of Banach spaces.

References

[1] E. D. Nursultanov. Multiparameter interpolation functors and Lorentz spaces $L_{p,\overline{q}}$, $\overline{q} = (q_1, \dots, q_n)$. *Functzionalnyi analis i ego pril.*, **31**(2):79–82, 1997.