International Conference on Analysis and Geometry
August 30 – September 3, 1999, Novosibirsk, Russia

Abstracts

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The Behaviour of a Free Boundary Problem Near the Prescribed Boundary

Suppose Ω is a domain in $\mathbf{R}^n$ and there are a function $u \in C^{1,1}(\overline{\Omega})$ and an open set $\mathcal{N} \subset \Omega$ such that

$$\begin{cases} Lu = f & \text{in } \mathcal{N}, \\ u = |\nabla u| = 0 & \text{on } \Omega \setminus \mathcal{N}, \\ u = \phi & \text{on } \partial\Omega. \end{cases}$$

Here L is a linear elliptic operator, $f \in C^{\alpha}(\overline{\Omega})$, $f \geq \text{const} > 0$. Under certain smoothness conditions on $\partial\Omega$, ϕ and the coefficients of L we study the behaviour of the free boundary $\Gamma = \partial\mathcal{N} \cap \Omega$ in a vicinity of points where Γ meets $\partial\Omega$. For the case of nonnegative u we prove that if $x^0 \in \partial\Omega \cap \overline{\Gamma}$ is such a contact point then $\partial N \in C^1$ in some neighborhood of x^0 provided $\phi = 0$ near x^0. Besides, the second derivatives of u are continuous on $\overline{\mathcal{N}} \cap B_\rho(x^0)$ for some $\rho > 0$. Similar results are obtained for the case where there are no sign restrictions on u under the additional assumptions that Γ is a Lipschitz graph over $\partial\Omega$ near x^0.