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Abstracts

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Noncompact Local Stable Foliations

We investigate local leaf stability of noncompact foliations.

Let F be a smooth foliation of an arbitrary codimension q of n-manifold M. We call a leaf L of F local stable if 1) there exists a saturated neighborhood W of L which is a fibered over L with the q-dimensional disk D^q as fibre type, the fibres of which are transversal to the leaves of the foliation; 2) the traces of these neighborhoods W form a basis for the topology of the fibres; 3) L is covered by all leaves from W.

This definition adds to Ehresmann's definition of a local stability leaf the above condition 3). As well known, for compact and Riemannian foliations 3) is a corollary of 1) and 2).

We call F a local stable foliation if every leaf of F is local stable.

Theorem 1. *Let F be a local stable foliation of M. Then* i) *the all holonomy groups of F are finite*; ii) *the leaf space M/F is a smooth* (*non-Hausdorff in general case*) *q-orbifold*; iii) *the graph of a local stable foliation is always Hausdorff and forms a local trivial fibration over the foliated manifold.*

If there exists a compact leaf L of a local stable foliation F, then F is a compact foliation.

As known, for compact F the local stability is equivalent to each condition i) and ii). We have constructed examples of noncompact local stable foliations with non-Hausdorff spaces of the leaves.

Theorem 2. *Let F be a local stable foliation of a simple connected manifold M. Then the existence of an Ehresmann connection for F is equivalent to the property of the leaf space to be Hausdorff.*

If one of this conditions is satisfied, then F is a simple foliation.

Applications to foliations consistent with geometric structures are considered.

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