MULTIDIMENSIONAL OSCILLATIONS OF COLD PLASMA AND THE EULER-POISSON EQUATIONS

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PARTIAL DERIVATIVE EQUATIONS AND THEIR APPLICATIONS

The Euler-Poisson equations

$$
\begin{aligned}
&\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{V}) = 0,\\
&\frac{\partial (n\mathbf{V})}{\partial t} + \operatorname{div}\,(n\mathbf{V}\otimes\mathbf{V}) = \; k\, n\nabla\Phi - \nabla p(n) - \nu n\mathbf{V},\\
&\Delta\Phi = n - n_0,
\end{aligned}
$$

n (density), $\mathbf{V}$ (velocity),
Φ (a force potential), $p(n)$ (pressure),
depend on the time t and the point $x \in \mathbb{R}^{\mathbf{d}}$, $\mathbf{d} \geq 1$,
$n_0 \geq 0$ is the density background, $\nu \geq 0$ – the friction coefficient.
Positive or negative k corresponds to the repulsive and attractive force.

The equations of hydrodynamics of electron plasma in the non-relativistic approximation in dimensionless quantities:

$$\frac{\partial n}{\partial t} + \operatorname{div}(n\mathbf{V}) = 0,$$
$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\,\mathbf{V} = -\mathbf{E} - [\mathbf{V}\times\mathbf{B}] - \frac{1}{n}\nabla p(n) - \nu\mathbf{V},$$
$$\frac{\partial \mathbf{E}}{\partial t} = n\mathbf{V} + \mathrm{rot}\,\mathbf{B}, \qquad \frac{\partial \mathbf{B}}{\partial t} = -\mathrm{rot}\,\mathbf{E}, \qquad \operatorname{div}\mathbf{B} = 0,$$

n and $\mathbf{V} = (V_1, V_2, V_3)$ are the density and velocity of electrons, $\mathbf{E} = (E_1, E_2, E_3)$ and $\mathbf{B} = (B_1, B_2, B_3)$ are vectors of electric and magnetic fields. All components of solution depends on $t \in \mathbb{R}_+$ and $x \in \mathbb{R}^3$.

$n_0 > 0,\ k < 0$

$p = 0$ (cold plasma)

A class of solutions depending only on the radius-vector of point $r = \sqrt{x_1^2 + x_2^2 + x_3^2}$, i.e.

$$\mathbf{V} = F(t, r)\mathbf{r}, \quad \mathbf{E} = G(t, r)\mathbf{r}, \quad \mathbf{B} = Q(t, r)\mathbf{r}, \quad n = n(t, r),$$

where $\mathbf{r} = (x_1, x_2, x_3)$.
It implies $\mathbf{B} \equiv 0$, $\mathrm{rot}\, \mathbf{E} = 0$.

Under the assumption that the solution is sufficiently smooth and that the steady-state density n_0 is equal to 1, we get

$$n = 1 - \operatorname{div} \mathbf{E},$$

therefore n can be removed from the system.

The resulting system is

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\, \mathbf{V} = -\mathbf{E} - \nu V, \qquad \frac{\partial \mathbf{E}}{\partial t} + \mathbf{V} \mathrm{div}\, \mathbf{E} = \mathbf{V}. \tag{1}$$

If we introduce the potential Φ such that $\nabla\Phi = -\mathbf{E}$, we can rewrite the system as the Euler-Poisson equations with $n_0 = 1$. Can be considered in any space dimensions **d**.

$$\frac{dV}{dt} = -E - 0 \cdot V, \qquad \frac{dE}{dt} = V, \qquad \frac{dx}{dt} = V.$$

It imlpies $V^2 + E^2 = V^2(0) + E^2(0) = \mathrm{const}$ along each of the characteristics $x = x(t)$.
For $v = V_x$, $e = E_x$ we get

$$\frac{dv}{dt} = -v^2 - e, \quad \frac{de}{dt} = (1-e)v, \quad e < 1,$$

$$v^2 + 2e - 1 = C(e-1)^2,$$

a second-order curve, its type depends on the sign of

$$\Delta = v^2 + 2e - 1.$$

if $\Delta(0) < 0$, then the phase curves is ellipse, the derivatives remain bounded for $t > 0$. Otherwise, the phase curve is a parabola for $\Delta(0) = 0$ or a hyperbola for $\Delta(0) > 0$, the derivatives become infinite.

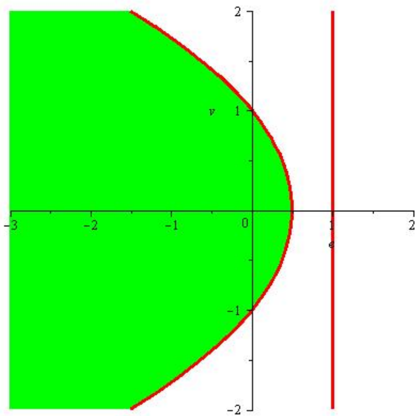


Figure: Criterion of smoothness, $\Delta(0) < 0$

Criterion of a singularity formation ([Chizhonkov, R, 2020], [Engelberg, Liu, Tadmor, 2001]

Theorem

For the existence and uniqueness of continuously differentiable $2\pi-$ periodic in time solution (V, E) of

$$
\begin{aligned}
\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} &= -E, \quad \frac{\partial E}{\partial t} + V\frac{\partial E}{\partial x} = V, \\
(V, E)|_{t=0} &= (V_0, E_0) \in C^2(\mathbb{R})
\end{aligned}
$$

it is necessary and sufficient that inequality

$$
\left(V_0'(x)\right)^2 + 2\,E_0'(x) - 1 < 0
$$

holds at each point $x \in \mathbb{R}$.
If there exists at least one point x_0 for which the opposite inequality holds, then the derivatives of the solution become infinite in a finite time.

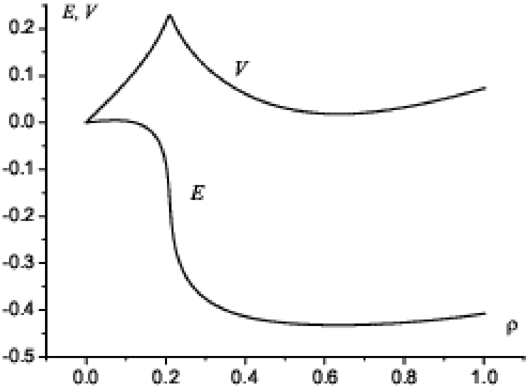


Figure: Spatial distribution of velocity and electric field near the moment of formation of singularity (by E.V.Chizhonkov).

$\nu > 0$ [R, Chizhonkov, Delova, 2021]

Phase plane (e, v):

- 1. $\nu = 0$: one equilibrium point $(0,0)$, a center;
- 2. $0 < \nu < 2$: one equilibrium point $(0,0)$, a stable focus;
- 3. $\nu = 2$: two equilibria, $(0,0)$, a degenerate stable node, $(1,-1)$, a saddle-node;
- 4. $\nu > 2$: three equilibria, $(0,0)$, a stable node, $(1, -\frac{1}{2}(\nu - \sqrt{\nu^2 - 4})$, a saddle, $(1, -\frac{1}{2}(\nu + \sqrt{\nu^2 - 4})$, an unstable node.

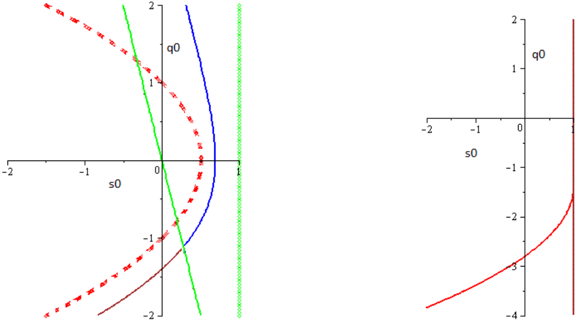


Figure: Left: $\nu < 2$ in comparison with $\nu = 0$ (dotted line). Right: $\nu > 2$.

$\nu = \nu(n)$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = -E - \nu(n)V, \quad \frac{\partial E}{\partial t} + V\frac{\partial E}{\partial x} = V, \quad n = 1 - \frac{\partial E}{\partial x},$$

$$(V, E)|_{t=0} = (V_0(x), E_0(x)) \in \mathcal{A}(\mathbb{R}).$$

For the prototypic function $\nu(n) = \nu_0 n^\gamma$ the threshold value is $\gamma = 1$. For $\gamma \geq 1$ the solution does not form the gradient catastrophe.

$\nu = \nu(n)$ [R, 2021]

Theorem

Let $f(n) \in \mathcal{A}(\mathbb{R}_+)$ *be a nonnegative function satisfying conditions*

$$\lim_{\eta\to\infty} \frac{\eta f'(\eta)}{f(\eta)} = \mathrm{const}$$

and

$$\int\limits_{\eta_0>0}^{+\infty} \frac{f(\eta)}{\eta^2}\, d\eta = \infty,$$

$\nu(n) = \epsilon f(n)$, $\epsilon = \mathrm{const} > 0$. *Under the assumption that the formation of singularity is associated with a gradient catastrophe (unboundedness of the first derivatives), the problem admits a global in time classical (C^1-smooth) solution. Otherwise, one can find the data such that the derivatives of solution blow up in a finite time.*

$\nu = 0,\ p(n) \neq 0$

Theorem

Assume $p(n) = \frac{1}{\gamma} n^\gamma$, $\gamma > 1$. *A continuously differentiable* $2\pi-$ *periodic in time solution* (V, E) *of*

$$\frac{\partial V}{\partial t} + V \frac{\partial V}{\partial x} = -E - \mu \frac{1}{n} \frac{\partial p(n)}{\partial x}, \quad \frac{\partial E}{\partial t} + V \frac{\partial E}{\partial x} = V,$$

$$n = 1 - \frac{\partial E}{\partial x},$$

$$(V, E)|_{t=0} = (V_0, E_0) \in C^2(\mathbb{R})$$

exists iff

$$\big(V_0'(x)\big)^2 + 2\, E_0'(x) - 1 < \mu \frac{(E_0''(x))^2}{(1 - E_0'(x))^{3-\gamma}}$$

holds at each point $x \in \mathbb{R}$.

The pressure generally does not remove or postpone a singularity.

The type of the singularity changes:

$\mu = 0$: V, E – the gradient catastrophe, n – strong singularity;

$\mu > 0$: V, n – the gradient catastrophe, E – weak singularity;

Example: $\gamma = 2$

$$\frac{\partial V}{\partial t} + V\frac{\partial V}{\partial x} = -E + \mu\frac{\partial^2 E}{\partial x^2},$$
$$\frac{\partial E}{\partial t} + V\frac{\partial E}{\partial x} = V,$$

A particular case of

$$\frac{\partial \mathfrak{V}}{\partial t} + V_1\frac{\partial \mathfrak{V}}{\partial x} = Q\mathfrak{V} + B\frac{\partial^2 \mathfrak{V}}{\partial x^2}, \qquad (2)$$

where $\mathfrak{V} = (V_1, V_2, \dots, V_n)$, $V_i = V_i(t,x)$, Q and B are $n \times n$ constant matrices.
Here $V_1 = V$, $V_2 = E$, $Q = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$, $B = \begin{pmatrix} 0 & \mu \\ 0 & 0 \end{pmatrix}$.

$B\frac{\partial^2 \mathfrak{V}}{\partial x^2}$ looks like a viscous term, however in fact its sense is different.

Consider the initial data

$$(\mathbf{V}, \mathbf{E})|_{t=0} = (F_0(r)\mathbf{r}, G_0(r)\mathbf{r}), \quad (F_0(r), G_0(r)) \in C^2(\bar{\mathbb{R}}_+),$$

where $\mathbf{r} = (x_1, \dots, x_{\mathbf{d}})$, $\quad r = |\mathbf{r}|$, with the physically natural condition $n|_{t=0} > 0$.

Definition

Solution $(\mathbf{V}, \mathbf{E})$ is called an affine solution if it has the form $\mathbf{V} = \mathfrak{V}(t)\mathbf{r}$, $\mathbf{E} = \mathfrak{E}(t)\mathbf{r}$, where $\mathfrak{V}$ and $\mathfrak{E}$ are $(\mathbf{d} \times \mathbf{d})$ matrices.

Definition

Solution $(\mathbf{V}, \mathbf{E})$ is called a simple wave if it has the form $\mathbf{V} = F(t, r)\mathbf{r}$, $\mathbf{E} = G(t, r)\mathbf{r}$, where $F(t, r)$ and $G(t, r)$ are functionally dependent.

Theorem

The solution of the Cauchy problem for $\mathbf{d} \geq 2$, $\mathbf{d} \neq 4$, *blows up in a finite time for all initial data, possibly except for the data, corresponding to simple waves.*

If the solution is globally smooth in time, then it is either affine or tends in the C^1*-norm to an affine solution as* $t \to \infty$.

Explicit solutions along characteristics

F and G satisfy the following Cauchy problem:

$$\frac{\partial G}{\partial t} + Fr\frac{\partial G}{\partial r} = F - \mathbf{d}FG, \quad \frac{\partial F}{\partial t} + Fr\frac{\partial F}{\partial r} = -F^2 - G,$$

$$(F(0,r), G(0,r)) = (F_0(r), G_0(r)), \quad (F_0(r), G_0(r)) \in C^2(\bar{\mathbb{R}}_+).$$

Along the characteristic

$$\dot{r} = Fr,$$

starting from the point $r_0 \in [0, \infty)$ system (3) takes the form

$$\dot{G} = F - \mathbf{d}FG, \qquad \dot{F} = -F^2 - G, \tag{3}$$

Thus,

$$\frac{1}{2}\frac{dF^2}{dG} = -\frac{F^2 + G}{1 - \mathbf{d}G},$$

which is linear with respect to F^2 and can be explicitly integrated.

For $\mathbf{d} = 2$

$$2F^2 = (2G-1)\ln|1-2G| + C_2(2G-1) - 1,$$
$$C_2 = \frac{1+2F^2(0,r_0)}{2G(0,r_0)-1} - \ln|1-2G(0,r_0)|,$$

for $\mathbf{d} = 1$ and $\mathbf{d} \geq 3$

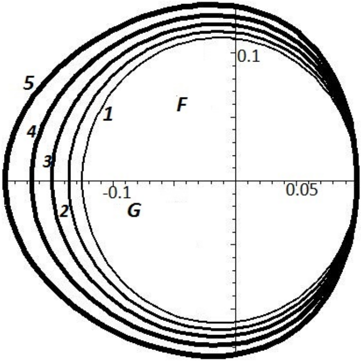
$$F^2 = \frac{2G-1}{\mathbf{d}-2} + C_{\mathbf{d}}|1-\mathbf{d}G|^{\frac{2}{\mathbf{d}}},$$
$$C_{\mathbf{d}} = \frac{1-2G(0,r_0)+(\mathbf{d}-2)F^2(0,r_0)}{(\mathbf{d}-2)|1-\mathbf{d}G(0,r_0)|^{\frac{2}{\mathbf{d}}}}.$$

Lemma

The period of revolution on the phase curve depends on $\mathbf{d}$ *and the starting point of trajectory, except for* $\mathbf{d} = 1$ *and* $\mathbf{d} = 4$, *where* $T = 2\pi$. *In the other cases the following asymptotics holds for the deviation of order* ε *from the origin:*

$$T = 2\pi(1 + \frac{1}{24}(\mathbf{d} - 1)(\mathbf{d} - 4)\varepsilon^2 + o(\varepsilon^2)), \quad \varepsilon \to 0,$$

i.e. for $\mathbf{d} \in (1, 4)$ *the period is less that* 2π, *for* $\mathbf{d} > 4$ *the period is greater that* 2π.



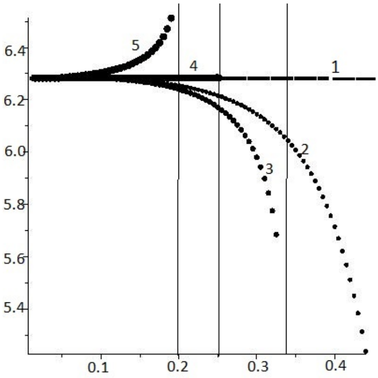


Figure: Phase portrait on the plane (F, G) for $G_+ = 0.1$ (left) and the dependence of the period on G_+ (right) for $\mathbf{d} = 1, 2, 3, 4, 5$.

The behavior of derivatives

Denote $\mathcal{D} = \operatorname{div} \mathbf{V}$, $\lambda = \operatorname{div} \mathbf{E}$.

$$\dot{\mathcal{D}} = -\mathcal{D}^2 + 2\,(\mathbf{d}-1)\,F\,\mathcal{D} - \lambda - (\mathbf{d}-1)\,\mathbf{d}\,F^2, \qquad \dot{\lambda} = \mathcal{D}\,(1-\lambda),$$

along the characteristic curve.
New variables: $u = \mathcal{D} - \mathbf{d}\,F$, $v = \lambda - \mathbf{d}\,G$:

$$\dot{u} = -u^2 - 2\,F\,u - v, \quad \dot{v} = -u\,v + (1 - \mathbf{d}\,G)\,u - \mathbf{d}\,F\,v. \qquad (4)$$

Linearization and the Radon lemma

Theorem (The Radon lemma)

A matrix Riccati equation

$$\dot{W} = M_{21}(t) + M_{22}(t)W - WM_{11}(t) - WM_{12}(t)W,$$

is equivalent to the homogeneous linear matrix equation

$$\dot{Y} = M(t)Y, \quad M = \begin{pmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{pmatrix}.$$

Let on some interval $\mathcal{J} \in \mathbb{R}$ *the matrix-function* $Y(t) = \begin{pmatrix} \mathfrak{Q}(t) \\ \mathfrak{P}(t) \end{pmatrix}$ *be a solution with the initial data*

$$Y(0) = \begin{pmatrix} I \\ W_0 \end{pmatrix}$$

Then $W(t) = \mathfrak{P}(t)\mathfrak{Q}^{-1}(t)$ *is the solution with* $W(0) = W_0$ *on* $\mathcal{J}$.

System (4) can be written as a matrix Riccati equation

$$W = \begin{pmatrix} u \\ v \end{pmatrix}, \quad M_{11} = \begin{pmatrix} 0 \end{pmatrix}, \quad M_{12} = \begin{pmatrix} 1 & 0 \end{pmatrix},$$
$$M_{21} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad M_{22} = \begin{pmatrix} -2\,F & -1 \\ 1 - d\,G & -d\,F \end{pmatrix}.$$

Thus, we obtain the Cauchy problem

$$\begin{pmatrix} \dot q \\ \dot p_1 \\ \dot p_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & -2\,F & -1 \\ 0 & 1 - d\,G & -d\,F \end{pmatrix} \begin{pmatrix} q \\ p_1 \\ p_2 \end{pmatrix}, \quad \begin{pmatrix} q \\ p_1 \\ p_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ u_0 \\ v_0 \end{pmatrix},$$

with periodical coefficients, known from (3).

The standard change of the variable $p_1(t) = P(t)\, e^{-\frac{\mathbf{d}+2}{2}\int\limits_0^t F(\tau)\, d\tau}$ reduces to

$$\ddot{P} + QP = 0, \quad Q = 1 - \frac{\mathbf{d}+2}{2} G - \frac{1}{4}(\mathbf{d}-2)(\mathbf{d}-4)F^2, \qquad (5)$$

$$q(t) = 1 + \int\limits_0^t p_1(\tau)\, d\tau = 1 + \int\limits_0^t P(\xi)\, e^{-\frac{d+2}{2}\int\limits_0^{\xi} F(\tau)\, d\tau}\, d\xi. \qquad (6)$$

Theorem implies that the solution of (4) blows up if and only if $q(t)$ vanishes at some point t_*, $0 < t_* < \infty$.

Idea of the proof: the Floquet theory

1. $Q(t)$ is periodic with period T, $Q(t) = Q(-t)$, therefore (5) has solutions

$$e^{\mu t}\mathcal{P}(t), \quad e^{-\mu t}\mathcal{P}(-t),$$

$\mathcal{P}$ is T-periodic, which can be taken as a fundamental system provided that μ is real.

2. Suppose $z(t)$ is a solution of (5) with initial conditions $z(0) = 1$, $z'(0) = 0$. Then

$$z(t) = \frac{1}{2\mathcal{P}(0)}\left(e^{\mu t}\mathcal{P}(t) + e^{-\mu t}\mathcal{P}(-t)\right),$$

Thus,

$$\cosh \mu T = z(T). \tag{7}$$

Unboundedness takes place for $|\cosh \mu T| > 1$.

If $z(T) > 1$, then $\mu \in \mathbb{R}$, and the general solution of (5) has the form

$$P = C_+ e^{\mu t} \mathcal{P}(t) + C_- e^{-\mu t} \mathcal{P}(-t).$$

Thus, for an arbitrary choice of the data P is unbounded and q oscillates with a growing amplitude.

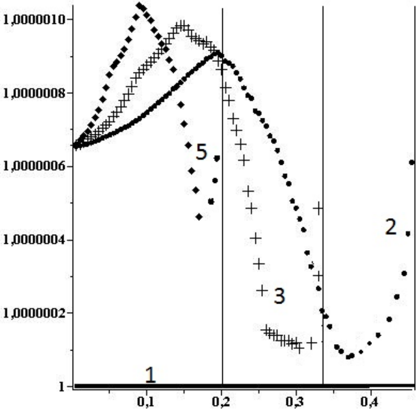


Figure: Dependence of $e^{\mu T}$ on G_+ for $\mathbf{d} = 1$ (and $\mathbf{d} = 4$ till $G_+ = 0.25$), solid line, $\mathbf{d} = 2$, solid circles, $\mathbf{d} = 3$, crosses, $\mathbf{d} = 5$, solid diamonds.

Simple waves

Theorem predicts the existence of non-affine solutions with special initial data, which are globally smooth and tends to an affine solution as $t \to \infty$.

To construct them, we look for simple waves $F = F(G)$. System (3) reduces to one equation

$$\frac{\partial G}{\partial t} + F(G) r \frac{\partial G}{\partial r} = F(G)(1 - \mathbf{d} G),$$

with $F(G)$ can be found from previous formulas, the periods of oscillations are equal for all characteristics. If we fix $C_{\mathbf{d}}$, we obtain the relation between G and F in this special kind of solution, and the corresponding initial data.

If G_r does not blow up, it tends to zero as $t \to \infty$ (E, V tend to the affine solution).

Configuration of solutions

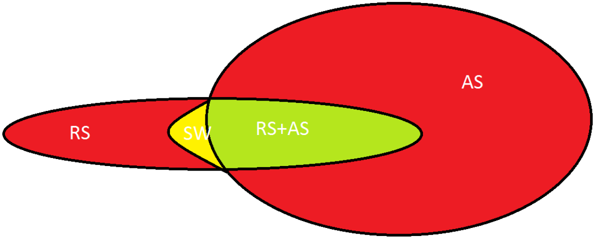


Figure: Affine solutions, radial solutions, simple waves.

$$\mathbf{V} = Q(t)\mathbf{r}, \qquad \mathbf{E} = R(t)\mathbf{r},$$

Q and R are $\mathbf{d} \times \mathbf{d}$ matrices with coefficients depending on t, $\mathbf{r}$ is the radius vector of the point $r \in \mathbb{R}^{\mathbf{d}}$.

E-P system reduces to a matrix system of ODE

$$\dot{Q} + Q^2 + R = 0, \quad \dot{R} - (1 - \operatorname{tr} R)Q = 0. \tag{8}$$

$\mathbf{d} = 3$, the oscillations in a plane perpendicular to e_3:

$$\mathbf{V} = Q\mathbf{r} = \begin{pmatrix} a(t) & b(t) & 0 \\ c(t) & d(t) & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{r}, \quad \mathbf{E} = R\mathbf{r} = \begin{pmatrix} A(t) & B(t) & 0 \\ C(t) & D(t) & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{r}.$$

Solutions with radial symmetry

$c = C = 0$, system (8) takes the form

$$\dot{a} = -A - a^2, \quad \dot{A} = a - 2Aa, \tag{9}$$

$a(t)$, $A(t)$ are periodic with period

$$T = 2\int\limits_{A_-}^{A_+} \frac{d\eta}{(1-2\eta)a(\eta)},$$

$A_- < 0$ and $A_+ > 0$ is the smaller and larger roots of the equation $a(A) = 0$. Besides, $\int\limits_0^T a(\tau)\, d\tau = 0$.

The period depends on $A(0) = \varepsilon$, $\varepsilon \in (0, \frac{1}{2})$ decreasing monotonically from 2π to $\sqrt{2}\pi$, and the asymptotic formula

$$T = 2\pi(1 - \frac{1}{12}\varepsilon^2 + o(\varepsilon^2)), \quad \varepsilon \to 0.$$

Arbitrary affine solution (2D)

$$\dot{A} = (1 - A - D)a, \qquad \dot{D} = (1 - A - D)d,$$
$$\dot{a} + a^2 + A = 0, \qquad \dot{d} + d^2 + D = 0.$$

To study the effect of deviation from symmetry, we make the substitution $d = a + \sigma$, $D = A + \delta$, which corresponds to the axisymmetric case for $\sigma = \delta = 0$:

$$\dot{A} = (1 - 2A)a - \delta a, \qquad \dot{a} = -a^2 - A,$$
$$\dot{\delta} = (1 - 2A - \delta)\sigma, \qquad \dot{\sigma} = -\sigma^2 - 2a\sigma - \delta.$$

Small perturbation from radial symmetry

We choose a small parameter ε and set

$$\begin{aligned} A(t) &= A_0(t) + \varepsilon^2 A_1(t) + o(\varepsilon^2), \quad a(t) = a_0(t) + \varepsilon^2 a_1(t) + o(\varepsilon^2), \\ \delta(t) &= \varepsilon^2 \delta_1(t) + o(\varepsilon^2), \quad \sigma(t) = \varepsilon^2 \sigma_1(t) + o(\varepsilon^2). \end{aligned}$$

For $\varepsilon = 0$ we obtain a globally smooth solution $A_0(t), a_0(t)$, which is a solution to system (9). For the functions A_1, a_1, δ_1, σ_1, discarding terms of the order of smallness $o(\varepsilon^2)$, we obtain the linear system

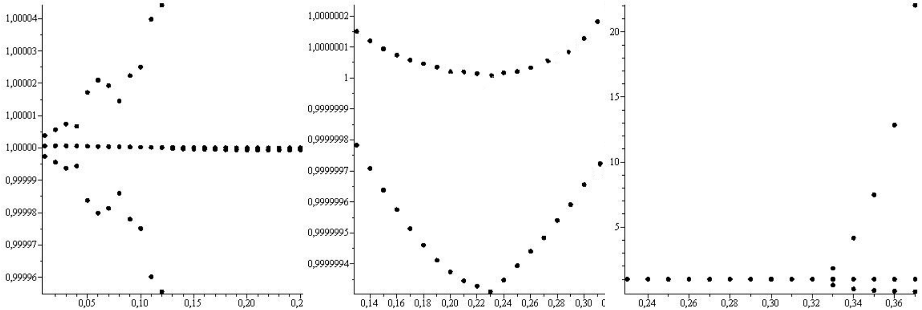
$$\begin{aligned} \dot{A}_1 &= -2a_0 A_1 + (1 - 2A_0)a_1 - a_0\delta_1, \quad \dot{a}_1 = -2a_0 a_1 - A_1, \\ \dot{\delta}_1 &= (1 - 2A_0)\sigma_1, \quad \dot{\sigma}_1 = -2a_0\sigma_1 - \delta_1. \end{aligned}$$

The Floquet theory for systems of linear equations with periodic coefficients

1. For the fundamental matrix $\Psi(t)$ ($\Psi(0) = E$) there exists a constant matrix M, possibly with complex coefficients, such that $\Psi(T) = e^{TM}$, where T is the period of the coefficients. The eigenvalues of the matrix of monodromy e^{TM} are called the characteristic multipliers of the system.

2. If among the characteristic multipliers there are such that their absolute value is greater than one, then the zero solution of the studied linear system is unstable in the sense of Lyapunov.

Characteristic multipliers in dependence on $A(0)$



Theorem

1. The zero equilibrium of system (8) *is unstable in the sense of Lyapunov in the class of affine solutions.*
2. Any small asymmetric affine perturbation of a globally smooth radially symmetric affine solution of system (8) *blows up in a finite time.*

$\mathbf{d} \geq 2$, $\nu = \mathrm{const} > 0$ [R, 2023]

$$\frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V} \cdot \nabla)\, \mathbf{V} = \; -\mathbf{E} - \nu \mathbf{V}, \quad \frac{\partial \mathbf{E}}{\partial t} + \mathbf{V} \operatorname{div} \mathbf{E} = \mathbf{V}.$$

$$(\mathbf{V}, \mathbf{E})|_{t=0} = (\mathbf{V}_0(r), \mathbf{E}_0(r)) = (F_0(r)\mathbf{r}, G_0(r)\mathbf{r}).$$

Theorem

For arbitrary small $\nu > 0$ *there exists* $\varepsilon(\nu) > 0$*, such that the solution of the problem satisfying*

$$\|\mathbf{V}_0(r), \mathbf{E}_0(r)\|_{C^1(\overline{\mathbb{R}}_+)} < \varepsilon,$$

keeps C^1 *- smoothness for all* $t > 0$*. Moreover,*

$$\|\mathbf{V}, \mathbf{E}\|_{C^1(\overline{\mathbb{R}}_+)} \le \mathrm{const}\, e^{-\frac{\nu}{2}t} \to 0, \quad t \to \infty.$$

Theorem

For arbitrary initial data (40) *there exists such* $\nu > 2$ *that the solution of problem* (10) - (40) *keeps* C^1 *- smoothness for all* $t > 0$ *and the asymptotic property*

$$\|\mathbf{V}, \mathbf{E}\|_{C^1(\overline{\mathbb{R}}_+)} \le \mathrm{const}\, e^{-\frac{\nu-\sqrt{\nu^2-4}}{2}t} \to 0, \quad t \to \infty.$$

holds.

References

1. Rozanova O. S., Chizhonkov E. V. On the conditions for the breaking of oscillations in a cold plasma, ZAMP, 72 (2021)

2. Rozanova O.S. Suppression of singularities of solutions of the Euler-Poisson system with density-dependent damping, Phys D: Nonlinear Phenomena, 429 (2022)

3. Chizhonkov E.V., Delova M.I., Rozanova O.S. High precision methods for solving a system of cold plasma equations taking into account electron-ion collisions, Russ J Num Anal Math Mod, 36, 3 (2021)

4. Rozanova O.S. On the behavior of multidimensional radially symmetric solutions of the repulsive Euler-Poisson equations, Phys D: Nonlinear Phenomena, 443 (2023)

5. Rozanova O.S, Turzynsky M.K. On the properties of affine solutions of cold plasma equations, Comm Math Sci 21 (2023)

6. Rozanova O.S. On the properties of multidimensional electrostatic oscillations of an electron plasma, Math Meth Appl Sci, 46, 6 (2023)

Thank you!