

LIBRATIONS, INSTANTONS, TUNNELING AND LOW BANDS OF 2-D SCHRÖDINGER OPERATOR FOR QUANTUM DIMERS

ANATOLY ANIKIN, SERGEY DOBROKHOTOV, MIKHAIL KATSNELSON

Librations are special unstable trajectories of n -D Hamilton systems with Hamiltonians $H = p^2 - V(x)$, $V(x) \geq 0$, coinciding the energy level curves for the potential $V(x)$ having several maxima in the case when the configuration space is Euclidian one or at least one maximum when the configuration space is a cylinder. Libration pass to singular solutions (called instantons) when the energy level tends to zero ($\max V$). It is well known that instantons appear in tunnel problems related to low parts of spectra of Schrödinger operators. We show that in spectral problems librations are much more natural and pragmatic objects then instantons. As example we discuss spectral semiclassical asymptotics for 2-D Schrödinger operator with the potential periodic in x and increasing at infinity in y , which describes quantum dimers.

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SERGEY YURIEVICH DOBROKHOTOV
A. ISHLINSKIY INSTITUTE FOR PROBLEMS IN MECHANICS,
101 VERNADSKY AVENUE, BLDG. 1, 119526, MOSCOW, RUSSIA

MOSCOW INSTITUTE OF PHYSICS AND TECHNOLOGY,
9 INSTITUTSKIY LANE, 141700, DOLGOPRUDNY, MOSCOW REGION, RUSSIA
E-mail address: dobr@ipmnet.ru

ANIKIN ANATOLY YURIEVICH,
A. ISHLINSKIY INSTITUTE FOR PROBLEMS IN MECHANICS,
101 VERNADSKY AVENUE, BLDG. 1, 119526, MOSCOW, RUSSIA

MIKHAIL KATSNELSON,
INSTITUTE FOR MOLECULES AND MATERIALS, RADBOUD UNIVERSITY OF NIJMEGEN
135 HEIJENDAALSEWEG, 6525AJ, NIJMEGEN, THE NETHERLANDS