ON PERIODIC ORBITS IN COMPLEX PLANAR BILLIARDS

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A conjecture of Victor Ivrii (1980) says that in every billiard with smooth boundary the set of periodic orbits has measure zero. This conjecture is closely related to spectral theory. Its particular case for triangular orbits was proved by M. Rychlik [1] (1989, in two dimensions), Ya. Vorobets [2] (1994, in any dimension) and other mathematicians (see references to [3]). The case of quadrilateral orbits in dimension two was treated in our joint work with Yu. Kudryashov [3] (2012). We study the complexified version of planar Ivrii's conjecture with reflections from a collection of planar holomorphic curves. We present the classification of complex counterexamples with four reflections and partial positive results (see, [4] and references therein). The recent one (unpublished yet) says that a billiard on one irreducible complex algebraic curve without too complicated singularities cannot have a two-dimensional family of periodic orbits of any period. The above complex results have applications to other problems on real billiards: Tabachnikov's commuting billiard problem and Plakhov's invisibility conjecture [4].

References

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