ON THE PRESCRIBED VALUES OF THE SPECTRUM OF SECTIONAL CURVATURE OPERATOR OF LOCALLY HOMOGENEOUS LORENTZIAN 3-MANIFOLDS

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Let (M, g) is n-dimensional locally homogeneous (pseudo)Riemannian manifold, R is the Riemannian curvature tensor of the metric g.

The Riemann curvature tensor R at any point $X \in M$ can be associated with the sectional curvature operator $\mathcal{R}\colon \Lambda^2_x M \to \Lambda^2_x M$, defined by equation

$$\langle X \wedge Y, \mathcal{R}(T \wedge V)\rangle_x = R_x(X, Y, T, V),$$

where $\langle \cdot, \cdot \rangle$ is the induced scalar product in the space $\Lambda^2_x M$, defined by the rule $\langle X_1 \wedge X_2, Y_1 \wedge Y_2\rangle_x = \det\left(g_x(X_i, Y_j)\right)$.

The problem of establishing connections between the topology and the curvature of (pseudo)Riemannian manifold is one of the important problems of (pseudo)Riemannian geometry. One of possible variants is to investigate the spectrum of different curvature operators. The spectrum of the Ricci curvature operator of left-invariant Riemannian metrics on Lie groups was studied by J. Milnor. In particular, he found the possible signature of spectrum of the Ricci operator in the case of three-dimensional Lie groups with left-invariant Riemannian metric [1]. O. Kowalski, S. Nikcevic solved the problem of the prescribed values of the spectrum of Ricci operator on three-dimensional metric Lie groups and for the three-dimensional locally homogeneous Riemannian spaces [2]. Further, similar results for the sectional curvature operator were obtained by D.N. Oskorbin, E.D. Rodionov, O.P. Khromova [3, 4].

The case of locally homogeneous Lorentzian manifolds was investigated by G. Calvaruso and O. Kowalski [5]. In particular, they proved, that three-dimensional connected, simply connected, complete locally homogeneous Lorentzian manifold is either

(1) isometric to a three-dimensional Lie group equipped with a left-invariant Lorentzian metric, or
(2) a Lorentzian space form $\mathbb{R}^3_1$, $\mathbb{S}^3_1$ or $\mathbb{H}^3_1$, or
(3) a direct product $\mathbb{R} \times \mathbb{S}^2_1$, $\mathbb{R} \times \mathbb{H}^2_1$, $\mathbb{S}^2 \times \mathbb{R}_1$ or $\mathbb{H}^2 \times \mathbb{R}_1$, or
(4) a space which admit local coordinates (t, x, y) such that, with respect to the local frame field $\left\{(\frac{\partial}{\partial t}), (\frac{\partial}{\partial x}), (\frac{\partial}{\partial y})\right\}$, the Lorentzian metric g is given by

$$g = \begin{pmatrix} 0 & 0 & 1 \\ 0 & \varepsilon & 0 \\ 1 & 0 & f(x,y) \end{pmatrix},$$

where $\varepsilon = \pm 1$ and $f(x, y) = x^2\alpha + x\beta(y) + \xi(y)$, for any constant $\alpha \in \mathbb{R}$ and any functions β, ξ.

In this paper we study the problem of the prescribed values of the sectional curvature operator $\mathcal{R}$ on three-dimensional locally homogeneous Lorentzian manifolds with the help of the above classification.

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