RANDOM WALKS ON GROUPS ACTING ON TREELIKE SPACES

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Random walks on groups is a subject at the intersection of probability and group theory. We are looking for new examples and approaches in this area by examining group actions on topological spaces and applying methods of topological dynamics. A specific class of spaces that turns out to be of interest here is the spaces with treelike structure: classical trees, $\mathbb{R}$-trees, dendrons, dendrites, dendritic spaces, etc.

A *dendritic space* is a connected topological space every two points of which are separated by a third one. (Subsets A and B of a topological space X are said to be *separated* by a subset C of X if $X \setminus C = P \cup Q$, where P and Q are disjoint open subsets of $X \setminus C$ such that $A \subset P$ and $B \subset Q$.)

Compact dendritic spaces are called *dendrons*, and metrizable dendrons are called *dendrites*.

Let G be a countable group and let μ be a probability measure on G. The (right) *random walk on G with distribution μ* (or, briefly, *μ-walk*) is the time-homogeneous Markov chain whose state space is G, the transition probabilities are given by $P(g,h) = \mu(g^{-1}h)$, and the initial distribution is concentrated at the identity of the group. Realizations of this process are called *paths* of the random walk. Let P_μ denote the associated Markov measure on the path space $G^{\mathbb{N}_0}$.

We obtain a series of new results of the following kind.

Theorem. *Let G be a countable group acting on a dendrite D. Assume that D is not an interval. Assume moreover that D contains no proper G-invariant connected compact subsets. Take a point $x \in D$ such that $D \setminus \{x\}$ is disconnected, and let $f\colon G \to D$ be the map sending g to $g(x)$. Let μ be a probability measure on G whose support generates G as a semigroup. Then f maps almost all paths of the μ-walk (that is, P_μ-almost all paths) to convergent sequences.*

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