KRICHEVER – NOVIKOV'S OPERATORS WITH POLYNOMIAL COEFFICIENTS

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A generic form of pairs of rank two commuting ordinary differential operators of orders 4 and 6 operators on the elliptic spectral curve was found by I.M. Krichever and S.P. Novikov [1]. P.G. Grinevich [2] found conditions when Krichever–Novikov operators have rational coefficients. We show that for any elliptic spectral curve there exists non-self-adjoint operators of orders 4 and 6 with polynomial coefficients.

Theorem 1. *For arbitrary spectral curve* Γ *defined by the equation*

$$w^2 = F(z) = z^3 + c_2 z^2 + c_1 z + c_0, \quad c_i \in \mathbb{C}$$

arbitrary $(z_0, w_0) \in \Gamma, w_0 \neq 0$ *and arbitrary integer* $n \geq 1$ *there are polynomials*

$$R = \delta_{2n+2} x^{2n+2} + \ldots + \delta_0, \quad P = \beta_n x^n + \ldots + \beta_0,$$

with $\delta_{2n+2} \neq 0$, $\beta_n \neq 0$, *such that the operator*

$$L_4 = \left(\partial_x^2 + R\right)^2 + (4w_0 P_x)\partial_x + \partial_x(4w_0 P_x) - 16F(z_0)P^2 + 4F'(z_0)P - \frac{1}{2}F''(z_0) + z_0$$

commutes with an operator L_6 *of order 6 with polynomial coefficients, and the spectral curve of* L_4, L_6 *is* Γ.

References

[1] Krichever I.M., Novikov S.P.,"Holomorphic bundles over algebraic curves and nonlinear equations", *Russian Mathematical Surveys*, Vol. 35, No. 6, 47–68 (1980).

[2] Grinevich P.G., "Rational solutions for the equation of commutation of differential operators", *Functional Analysis and Its Applications*, Vol. 16, No. 1, 15–19 (1982).

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