Energy function for 3-diffeomorphisms with one-dimensional surface attractor and repeller

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The Lyapunov function was introduced by A.M. Lyapunov to study the stability of equilibrium states of differential equations. A smooth Lyapunov function in which the set of critical points coincides with the chain-recurrent set of the system is called the energy function. For example, such functions always exist for flows, whereas examples of cascades are known which do not have an energy function. The first example of such diffeomorphism was built on the 3-sphere by Pixton [1] in 1977 based on the wild arc of Artin-Fox [2]. The more surprising is the fact that there exists an energy function for cascades with chaotic dynamics, discovered in 2015 by V.Z. Grines, M.K. Barinova and O.V. Pochinka [3]–[5]. They established the existence of the energy function for surface Ω-stable diffeomorphisms with one-dimensional basis sets and for rough three-dimensional diffeomorphisms with surface basis sets and for rough 3-diffeomorphisms with surface basic sets. Ω-stable 3-diffeomorphisms the nonwandering set of which consists of exactly one surface attractor and one surface repeller are considered and the existence of the energy function of such diffeomorphisms is proved.

References

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