On the topology of manifolds admitting cascades attractor - repeller of the same dimension

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Let $f\colon M^n \to M^n$ be orientation preserving diffeomorphism of a smooth closed orientable manifold M^n satisfying axiom A of S. Smale (non-wandering set $NW(f)$ is hyperbolic and the set of periodic points is dense in $NW(f)$).

According to S. Smale's spectral theorem, the nonwandering set $NW(f)$ can be decomposed into a finite union of disjoint closed invariant sets (called basic sets), each of which contains a dense orbit.

It is well known that if the non-wandering set of diffeomorphism f consists of exactly two fixed points: a source and a sink, then the manifold M^n is diffeomorphic to the n-dimensional sphere S^n. If the dimension of a basic set of diffeomorphism f coincides with the dimension of the ambient manifold, then f is Anosov diffeomorphism, the basic set is an attractor and a repeller simultaneously and coincides with the manifold M^n. It was shown by J. Franks and Sh. Newhouse in the case when the dimension of a stable or unstable manifold of a periodic point of Anosov diffeomorphism is 1, that manifold M^n is diffeomorphic to the torus of dimension n (see [1] , [2]).

The report describes the results obtained in the works of V. S Grines, E. V. Zhuzhoma, Yu. a. Levchenko, V. Medvedev, O. Pochinka (see [3] - [5]), from which follows topological classification of manifolds M^n, admitting diffeomorphisms f whose nonwandering sets consist of an attractor and a repeller of the same dimension. In addition, we give sufficient conditions when the nonwandering set of the diffeomorphism f cannot consist of two basic sets of the same dimension.

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Bibliography

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