Topological conjugacy of gradient-like flows on n-dimensional sphere

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Gradient-like flows are continuous dynamical systems whose non-wandering set consists of a finite number of hyperbolic fixed points. Their invariant manifolds cross each other transversally.

Depending on research goals there are two important things: qualitative behaviour of the system (i.e. partition of a manifold into trajectories) and moving along the trajectories by the time. In dynamical systems theory *topological equivalence* is an existence of a homeomorphism sending trajectories of one flows into trajectories of another one preserving direction of moving; if such a homeomorphism preserves time of moving along the trajectories, then it is called *topological conjugacy* of flows. Searching for invariant determining topological equivalence class for a system is *topological classification.*

The non-wandering set is a finite. Hence, the problem of topological classification may be reduced to a combinatorial one. First time it was done by E. Leontovich and A. Mayer in [2], [3] for classification of flows with finite number of singular trajectories on 2-dimensional sphere. These results were developed in researches by M. Peixoto [5], A. Oshemkov, V. Sharko [4], S. Pilyugin [6], A. Prishlyak [7], where similar problem was solved for Morse-Smale flows on closed manifolds of dimensions 2,3 and higher. These works were dedicated to topological equivalency. In [1] there is proved that topological equivalent flows on surfaces are also conjugate, hence, all equivalence results are also true for conjugacy. In our work we obtained similar result for class G of gradient-like flows without heteroclinic trajectories on n-sphere, $n \geq 3$. Besides, we introduce topological combinatorial invariant for such flows, i.e. *bi-colour graph* and prove that two flows from G are topological conjugate iff their bi-colour graphs are isomorphic.

Acknowledgements. The work was done in collaboration with O. Pochinka and D. Malyshev with support of Russian Science Foundation, project No 17-11-01041.

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