Hurwitz numbers and matrix integrals labeled with chord diagrams

Orlov A.Yu.

Abstract: We shall consider the product of complex random matrices from the independent complex Ginibre ensembles. The product includes complex matrices $Z_i, Z^\dagger_i, i=1,\dots,n$ and $2n$ sources (complex matrices C_i and C_{*i}). Any such product can be represented by a chord diagram that encodes the order of the matrices in the product. We introduce the Euler characteristic E_* of the chord diagram and show that the spectral correlation functions of the product generate Hurwitz numbers that enumerate nonequivalent branched coverings of Riemann surfaces of genus g_*. The role of sources is the generation of branching profiles in critical points which are assigned to the vertices of the graph drawn on the base surface obtained as a result of gluing of the $2n$-gon related to the chord diagram in a standard way. Hurwitz numbers for Klein surfaces may also be obtained by a slight modification of the model. Namely, we consider $2n+1$ polygon and consider pairing of the extra matrix Z_{2n+1} with a "Mobius" tau function. Thus, the presented matrix models labelled by chord diagrams generate Hurwitz numbers for any given Euler characteristic of the base surface and for any given set of ramification profiles.