Vector Bundles and Difference Equations in a Complex Domain

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Applications of the theory of holomorphic vector bundles with meromorphic connections to the classical Riemann–Hilbert problem are well known. We are going to apply holomorphic vector bundles with meromorphic additive shift or q-shift to studying of generalized Riemann–Hilbert–Birkhoff problem for difference and q-difference systems.

As the application of this approach we obtain a generalization of Birkhoff's existence theorem and reprove the local existence theorem. We prove that for any admissible set of characteristic constants and monodromy there exists a system

$$Y(z + 1) = A(z)Y(z) \quad \text{or} \quad Y(qz) = Q(z)Y(z), \tag{1}$$

which has the given monodromy and characteristic constants.