Differential Equations with a Small Parameter and Multipeak Self-Oscillations

G.A. Chumakov and N.A. Chumakova (Novosibirsk)

Critical phenomena – such as hysteresis of steady states, kinetic and thermokinetic self-oscillations of the chemical transformation rate, and regular and chaotic self-oscillations – are observed in various catalytic systems (for example, see [1–3]). The nonlinear effects in the dynamics of heterogeneous catalytic reactions were intensively studied since the last third of the twentieth century.

Under study is a nonlinear dynamical system of autonomous ordinary differential equations with two fast variables x and y and one slow variable z. The equation for z contains a small parameter μ, and for $\mu = 0$ the system of *fast motions* is included in a one-parameter family of two-dimensional subsystems with parameter z [2–5]. Let us assume that each subsystem has a rough periodic solution l_z and, moreover, the complete system has a rough periodic solution L which, as $\mu \to 0$, tends to the periodic solution l_{z_0} for some $z = z_0$.

Taking a plane (y, z) transversal to L, we construct a point Poincaré map and prove the existence of an invariant manifold for the steady point corresponding to the periodic solution L. Note that L has an invariant manifold on a guaranteed interval with respect to y, and this interval is separated from zero as $\mu \to 0$. The proved theorem allows us to give some sufficient conditions for the existence and absence of multipeak self-oscillations in the dynamical system under consideration.

As an example, we consider a kinetic model of the heterogeneous catalytic reaction of hydrogen oxidation over nickel.

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