Classification of Bedford-McMullen carpets that are dendrites

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Definition: Let $\mathcal{S} = \{S_1, \dots, S_l\}$ be a system of (affine) contraction similarities in $\mathbb{R}^n$. A non-empty compact set K satisfying the equation $K = \bigcup\limits_{i=1}^{l} S_i(K)$ is called *the attractor of the system* $\mathcal{S}$.

We denote by $I = \{1, \dots, l\}$ the set of indices of the system $\mathcal{S}$, then $I^* = \bigcup\limits_{n=1}^{\infty} I^n$ denotes the set of all words $\mathbf{i} = i_1 \dots i_n$ of finite length in the alphabet I, called *multiindices*. We will use the notation $S_{\mathbf{j}} = S_{j_1 j_2 \dots j_n} = S_{j_1} S_{j_2} \dots S_{j_n}$, and denote the $S_{\mathbf{j}}(K)$ by $K_{\mathbf{j}}$.

Critical set of the attractor K of the system $\mathcal{S}$ is the set $C := \{x : \ x \in S_i(K) \cap S_j(K),\ S_i, S_j \in \mathcal{S}\}$ of points of pairwise intersections of copies of K. The set ∂K of all $x \in K$ such that for some $\mathbf{j} \in I^*$, $S_{\mathbf{j}}(x) \in C$ is called the *self-similar boundary* of the set K.

A *dendrite* is a locally connected continuum containing no simple closed curve.

Let K be a self-similar dendrite possessing finite self-similar boundary ∂K. The minimal subdendrite $\hat{\gamma} \subset K$, containing ∂K is called the *main tree* of the dendrite K.

Definition 1. *Let a set $K = K(\mathcal{S})$ be a self-similar continuum which possesses one-point intersection property.* The intersection graph $\Gamma(\mathcal{S})$ *of the system $\mathcal{S}$ is a bipartite graph with parts $\mathcal{K} = \{K_i : \ i \in I\}$ and $\mathcal{P} = \{p : \ p \in K_i \cap K_j,\ i, j \in I, i \neq j\}$, and with a set of edges $E = \{(K_i, p) : p \in K_i\}$.*

Definition 2. *Let $D = \{d_1, \dots, d_l\} \subset \{0, 1, \dots, n-1\} \times \{0, 1, \dots, m-1\}$, where $n, m \geq 2$, and $n, m \in \mathbb{N}$.* A Bedford-McMullen carpet *with a generating matrix $M = diag(n, m)$ and a* digit set *D is a self-affine set $K \subset \mathbb{R}^2$, satisfying the equation $K = \bigcup\limits_{i=1}^{l} S_i(K)$, where $S_i(z) = M^{-1}(z + d_i)$.*

A way was found to express the self-similar boundary of the Bedford-McMullen carpet and make sure that this boundary is finite. Now we can prove and use the following theorem.

Theorem 1. *A Bedford-McMullen carpet K is a dendrite iff its bipartite intersection graph is a tree.*

This theorem allows us to construct an algorithm showing whether a Bedford-McMullen carpet is a dendrite.

Finally, the following result allows us to classify these Bedford-McMullen carpets that are dendrites.

Theorem 2. *The Bedford-McMullen carpets dendrites may be divided to 7 types depending on the topology of their main tree.*

References

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