On rigorous justification of the wave turbulence theory in the Zakharov-L'vov stochastic setting

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The wave turbulence theory (WT) has been intensively developed in physical works since the 1960's, while mathematical works devoted to its rigorous justification started to appear only in the last decade, and the problem still remains poorly understood. WT can be viewed as a kinetic theory of interacting nonlinear waves, parallel to the famous R. Peierls' kinetic theory, and also as a toy model for the strong turbulence theory.

From the mathematical point of view, WT is a heuristic approach for studying small amplitude solutions to nonlinear Hamiltonian PDEs with periodic boundary conditions of large period. The fundamental assertion of WT is that distribution over the Fourier frequencies of the total energy of a solution is governed by a nonlinear kinetic equation, called the *wave kinetic equation* and going back to R. Peierls.

I will talk about my the joint works [1-3] with S.B. Kuksin, as well as with S.G. Vleduts and A. Maiocchi, in which we completed the first step in a rigorous justification of this assertion for a solution to the nonlinear Schrödinger equation subject to a random perturbation. Our result heavily uses tools from analytic number theory [4].

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References

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