Model of Josephson junction, dynamical systems on $\mathbb{T}^2$, determinantal surfaces and Painlevé 3 equations.

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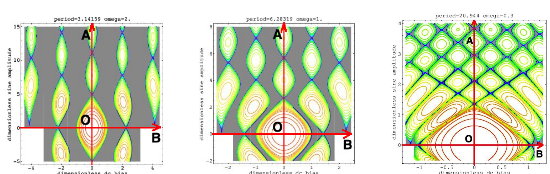
The tunneling effect predicted by B.Josephson (Nobel Prize, 1973) concerns the *Josephson junction:* two superconductors separated by a very narrow dielectric. It states existence of a supercurrent through it and equations governing it. The *overdamped* Josephson junction is modeled by a family of differential equations on 2-torus depending on 3 parameters: B (abscissa), A (ordinate), ω (frequency). We study its *rotation number* $\rho(B, A; \omega)$ as a function of (B, A) with fixed ω. The *phase-lock areas* are those level sets $L_r := \{\rho(B, A) = r\} \subset \mathbb{R}^2$ that have non-empty interiors. They exist only for integer rotation number values r: this is the rotation number quantization effect discovered by V.M.Buchstaber, O.V.Karpov and S.I.Tertychnyi. They are analogues of the famous Arnold tongues. Each L_r is an infinite chain of domains going vertically to infinity and separated by points called *constrictions* (expect for those with $A = 0$). See the phase-lock area portraits below for $\omega = 2,\ 1,\ 0.3$.



In a joint work of Yu.P.Bibilo and the speaker it was shown that *all constrictions in L_r lie in the vertical line $\Lambda_r := \{B = \omega r\}$.*

Buchstaber, Karpov and Tertychnyi discovered an equivalent description of the model by linear systems of differential equations on $\overline{\mathbb{C}}$. As was shown by Buchstaber and Tertychnyi, these systems are equivalent to special double confluent Heun equations. They described the parameter loci of Heun equations with polynomial solutions: the so-called *determinantal curves.*

We give a 4-dimensional extension of the model, whose parameter space is foliated by isomonodromic families of linear systems via Painlevé 3 equation. Using results on structure of the determinantal surfaces (leaf-saturated extensions of the determinantal curves), we prove genus formula for the curves (Netay conjecture, our joint result). We give a survey of results and problems.