Semiclassical dynamics of quasiparticles governed by the nonlocal nonlinear Schrödinger equation with an anti-Hermitian term

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The method of semiclassically concentrated states (MSCS) based on ideas of the Maslov complex germ method [1] allows one to construct asymptotic solutions to the nonlocal nonlinear Schrödinger equation that are localized in a neighbourhood of a trajectory in the phase space [2]. In this work, we consider the more complex problem of constructing solutions that are localized in a neighbourhood of multiple trajectories rather than a single one. We treat such solutions as semiclassical quasiparticles that has their own trajectories and interact with each other due to the laws governed by the nonlocal nonlinear Schrödinger equation (NNLSE).

We deal with the Cauchy problem for the following NNLSE with an anti-Hermitian term:

$$\Big\{ -i\hbar\partial_t + H(\hat{z},t)[\Psi] - i\hbar\Lambda\breve{H}(\hat{z},t)[\Psi]\Big\}\Psi(\vec{x},t) = 0, \quad \Psi(\vec{x},0) = \psi(\vec{x}),$$

$$H(\hat{z},t)[\Psi] = V(\hat{z},t) + \varkappa\int\limits_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)W(\hat{z},\hat{w},t)\Psi(\vec{y},t), \qquad (1)$$

$$\breve{H}(\hat{z},t)[\Psi] = \breve{V}(\hat{z},t) + \varkappa\int\limits_{\mathbb{R}^n} d\vec{y}\,\Psi^*(\vec{y},t)\breve{W}(\hat{z},\hat{w},t)\Psi(\vec{y},t),$$

where $\vec{x},\vec{y}\in\mathbb{R}^n$, $\hat{\vec{p}} = -i\hbar\partial_x$, $\hat{\vec{p}}_y = -i\hbar\partial_y$, $\hat{z} = (\hat{\vec{p}},\vec{x})$, $\hat{w} = (\hat{\vec{p}}_y,\vec{y})$, and $\hbar$ is a formal small parameter. The operators $V(\hat{z},t)$, $\breve{V}(\hat{z},t)$, $W(\hat{z},\hat{w},t)$, and $\breve{W}(\hat{z},\hat{w},t)$ are pseudo-differential operators with smooth symbols.

The generalization of the MSCS to the NNLSE with an anti-Hermitian term was proposed in [3]. In this work, we focus on constructing asymptotic solutions to (1) associated with the dynamical system of ordinary differential equations describing the semiclassical dynamics of quasiparticles. Once such system is solved, the respective asymptotic solutions to (1) can be found explicitly.

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References

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