Fast sixth-order algorithm based on the generalized Cayley transform for the Zakharov-Shabat system

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The nonlinear Schrödinger equation (NLSE) is widely used in telecommunication applications, since it allows one to describe the propagation of pulses in an optical fiber. Recently some new approaches based on the nonlinear Fourier transform (NFT) have been actively explored to compensate for fiber nonlinearity and to exceed the limitations of nonlinearity-imposed limits of linear transmission methods. Despite the fact that the numerical solution of NLSE is a general problem, nevertheless, the optical community has been focusing on this issue. Improving the accuracy of the NFT algorithms remains an urgent problem in optics. In particular, it is important to increase the approximation order of the methods, especially in problems where it is necessary to analyze the structure of complex waveforms. To correctly describe them and their spectral parameters, more accurate and fast numerical methods are needed.

We propose a novel general approach for constructing sixth-order (with respect to an integration step) finite-difference schemes for first-order linear differential systems [1]. These schemes are based on the generalized Cayley transform and include exponential integrators as a special case. If the system has a time-dependent skew-hermitian matrix then the schemes conserve the quadratic first integral automatically [2].

Then we apply our method to solve the direct spectral problem for the Zakharov-Shabat system. New schemes with fractional rational transition matrix allow the use of fast algorithms to solve the initial problem for a large number of values of the spectral parameter.

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