On the restricted SIS-model with migration over a spatially heterogeneous profile

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A typical approach of mathematical epidemiology to describing the spread of a pandemic is to consider point models (see [1] and references therein). Let one suppose that there is a quite large group of people, and some of these people have an infectious disease that is transmitted through contact between healthy and sick people at a β rate. Thus, this process is described by a restricted SIS-model [2].

But in practice such group can migrate along the x-axis at a variable velocity $V(x)$, expressing cross-country motion of this people. It means that this situation will be described by the following distributed dynamical system:

$$\frac{\partial S}{\partial t} + V(x)\,\frac{\partial S}{\partial x} = -\beta\, S\, I\,, \qquad \frac{\partial I}{\partial t} + V(x)\,\frac{\partial I}{\partial x} = \beta\, S\, I\,, \tag{1}$$

where $S(x,t)$ is linear density of susceptible to this disease individuals and $I(x,t)$ is linear density of infectious ones.

System (1) ought to be provided by the next initial conditions:

$$S(x,0) = S_0(x) \geq 0\,, \qquad I(x,0) = I_0(x) \geq 0\,, \qquad x \in \mathbb{R}\,. \tag{2}$$

In the report within the framework of the method of characteristics [3] exact solution of the Cauchy problem (1)-(2) has been obtained.

Moreover, in the report explicit expressions of these exact solutions for the following functions:

$$V(x) = \frac{V_m}{\cosh(x/L)} \tag{3}$$

and

$$V(x) = V_m \left[1 + \frac{1}{1 + \exp(x/L)}\right] \tag{4}$$

has been done.

In dependencies (3) and (4) V_m is maximal velocity of movement and L is the characteristic spatial scale of the velocity change.

For functions (3) and (4) the report provides graphs of functions $S(x,t)$ and $I(x,t)$ for various moments of time for epidemiologically realistic initial conditions (2).

This research seems to be useful for the estimation of the effectiveness of quarantine measures to protect the population of different countries from the penetration of pandemics across state borders.

References

[1] S.I. Kabanikhin, O.I. Krivorotko, *Optimization methods for solving inverse immunology and epidemiology problems*, Comp. Mat. Math. Phys, **60**:4 (2020), 580–589.

[2] F. Brauer, P. Driessche, J. Wu, eds., *Mathematical Epidemiology*, Springer Berlin, Heidelberg, 2008.

[3] L.C. Evans, *Partial Differential Equations*, American Mathematical Society, Providence, 1998.