Numerical splitting schemes for solving the Ginzburg-Landau equation

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Two new numerical algorithms have been developed to simulate the dynamics of an optical signal in optical fibers. The general characteristics of an optical signal as a result of generation in a resonator can be described using a dynamic model based on the complex cubic Ginzburg-Landau equation (GLE), which takes into account the saturated gain and dissipative terms responsible for the distributed action of various intracavity devices [1].

The GLE is a nonlinear partial differential equation, which in general cannot be solved analytically. One of the most widely used methods for the numerical solution of such equations is the split-step Fourier method [2]. As a rule, the equation is splitted into linear and nonlinear steps and then the Strang decomposition [2] is used, which allows one to obtain second order accuracy in the evolutionary variable. The GLE contains some terms in addition to dispersion and nonlinearity, their influence also must be taken into account in the numerical algorithm. An additional complication is that the coefficients of the equation include terms that depend on the energy of the optical field.

The paper proposes two new effective modifications of the split-step Fourier method for the numerical solution of equations of this type. The first algorithm is based on the application of a new way of separating the physical processes affecting the optical signal during propagation in a fiber, which made it possible to express the action of both nonlinear and dispersive spatial steps by explicit analytical expressions. The second proposed method allowed us to significantly improve the accuracy of calculations by taking into account the evolution of energy in the coefficients of the equation. Numerical experiments have shown that the new schemes make it possible to obtain the second order of approximation with respect to the evolutionary variable, in contrast to the classical scheme, which provides only the first order of approximation. The analysis of numerical schemes was carried out based on a comparison of the numerical solution of the GLE with the analytical stationary solution.

Thus, the proposed modifications of the split-step Fourier method, taking into account the evolution of energy, make it possible to significantly increase the accuracy of solving the complex cubic Ginzburg-Landau equation. Increasing the accuracy will be especially useful for equations with variable coefficients, for which a correct description of the evolution of energy is even more important [3].

Thanks. This work is supported by the grant of Russian Science Foundation (Project No. 22-11-00287, https://rscf.ru/en/project/22-11-00287/) carried out in Federal Research Center for Information and Computational Technologies.

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