BIPARTITE GRAPHS, GENERATING SELF-SIMILAR DENDRITES

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Let $\mathcal{S} = \{S_1, \ldots, S_m\}$ be a system of contracting similarities in $\mathbb{R}^n$. A non-empty compact set K satisfying the equation $K = \bigcup\limits_{i=1}^{m} S_i(K)$ is called *the attractor of the system* $\mathcal{S}$.

Let $I = \{1, \ \ldots, \ m\}$ b the set of indices of the system $\mathcal{S}$, and $I^* = \bigcup\limits_{n=1}^{e} \infty I^n$ be the set of all finite words $\mathbf{i} = i_1 \ldots i_n$ in the alphabet I, so $S_{\mathbf{j}} = S_{j_1 j_2 \ldots j_n} = S_{j_1} S_{j_2} \ldots S_{j_n}$, and we denote the $S_{\mathbf{j}}(K)$ by $K_{\mathbf{j}}$.

The critical set of the attractor K of the system $\mathcal{S}$ is the set $C := \{x : \ x \in S_i(K) \cap S_j(K), \ S_i, S_j \in \mathcal{S}\}$. The set ∂K of all $x \in K$ such that for some $\mathbf{j} \in I^*$, $S_{\mathbf{j}}(x) \in C$ is called the *self-similar boundary* of the set K.

A *dendrite* is a locally connected continuum containing no simple closed curve.

Let K be a self-similar dendrite possessing finite self-similar boundary ∂K in this case K possesses one point intersection property. The minimal subdendrite $\hat{\gamma} \subset K$, containing ∂K is called the *main tree* of the dendrite K.

Consider *the intersection graph* $\Gamma(\mathcal{S})$ of the system $\mathcal{S}$ which is a bipartite graph with parts $\mathcal{K} = \{K_i : \ i \in I\}$ and $\mathcal{P} = \{p : \ p \in K_i \cap K_j, \ i, j \in I, i \neq j\} \cup \partial K$, and with a set of edges $E = \{(K_i, p) : p \in K_i\}$. There is a natural map φ of the set E to ∂K defined by $\varphi((K_i, p)) = S_i^{-1}(p)$, which defines the edge-marking on the graph $\Gamma(\mathcal{S})$. The pair $(\Gamma(\mathcal{S}), \varphi) = (\mathcal{K}, \mathcal{P}, E, \varphi)$ is called the sprout of the dendrite K, or k-sprout, where $k = \#\partial K$. Two k-sprouts (Γ_1, φ_1), (Γ_2, φ_2) are isomorphic if there is an isomorphism of graphs Γ_1 and Γ_2, compatible with φ_1 and φ_2.

Theorem 1. *Two self-similar dendrites are isomorphic iff their sprouts are isomorphic.*

We define a composition of k-sprouts and prove the following theorem:

Theorem 2. *The set of k-sprouts is a semigroup with unity.*

References

[1] Tetenov, A., *Finiteness properties for self-similar continua*// arXiv:2003.04202 (2021)

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