Construction of billiard with alternating slipping V. N. Zav'yalov (Moscow)

In the work [1] A.T.Fomenko introduced a new class of billiards. Let the material point move along a straight line inside a circle unless it hits the boundary at the point x. Rotate the radius-vector of the point x by fixed angle to get point y. Let us continue the motion of material point starting from the point y along the straight line that forms the same angle with the boundary. In this case, the direction of the trajectory "by" or "against" the clockwise direction is preserved. In other words, its continuation leaves a new point on the boundary at the same angle, "slipping" along the border. So such a class of systems was called "billiards with slipping".

In the previouse author's work on circular billiards with slipping the Fomenko-Zieschang invariants and isoenergy 3-manifolds were obtained. In this case, these manifolds are homeomorphic to lens spaces.

In our talk we will discuss one generalization of the notion of slipping. Let the radius-vector of the point x rotate depending on the angle of incidence. At singular values of the angle of incidence, each value of the function should be equal $p\pi/q$ for some rational p/q. Construction of such billiard systems, their basic properties and applications to the modeling of several mechanical systems will be discussed.

Thanks. This work is supported by the Russian Science Foundation (project 22-71-00111). The author is a scholarship holder of the "Basis" foundation.

References

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