Mean field control problem for mathematical model of information diffusion in online social networks

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The process of information diffusion in online social networks can be described in terms of mean-field games. We consider a large number of users who can take states $x \in [0,1]$, where 0 means that the user is involved in the process of information dissemination, and 1 means the opposite. Then the density of users $u(x,t) : [0,1] \times [1,T] \to \mathbb{R}$ obeys the Kolmogorov (Fokker—Plank) equation

$$(1) \qquad u_t + \nabla(u\alpha) + \left(\frac{u}{K_{cap}} - 1\right) r(t)u - Du_{xx} = 0, \quad x \in [0,1], \quad t \in [1,T],$$

with initial condition and boundary conditions of Robin type

$$(2) \qquad u(x,1) = \varphi(x), \quad x \in [0,1]$$

$$(3) \qquad \beta_1 u_x(0,t) - \beta_2 u(0,t) = 0, \quad u_x(1,t) = 0, \quad t \in [1,T].$$

For $\alpha = 0$ equation (1) is investigated in [1].

Here $\alpha(x,t) : [0,1] \times [1,T] \to \mathbb{R}$ is the control parameter (or, in other words, user's strategy) to ensure the Nash equilibrium of a system of interacting agents and minimize the value functional in respect to (u,α)

$$J(u,\alpha) = \int_1^T \int_0^1 \left(d_1 e^{-t} \frac{u\alpha^2}{2} + d_2 (x-1)^2 u\right) dxdt.$$

Here d_1 and d_2 are weight coefficients.

Using the Lagrange multiplier method [2] a system similar to the Hamilton—Jacobi—Bellman equation is constructed

$$(4) \qquad \begin{cases} v_t + \alpha v_x + \left(1 - \dfrac{2u}{K_{cap}}\right) r(t)v + Dv_{xx} = -d_1 e^{-t} \dfrac{\alpha^2}{2} - d_2 (x-1)^2, \\ v(x,T) = 0, \quad x \in [0,1], \\ v_x(0,t) = v_x(1,t) = 0, \quad t \in [1,T]. \end{cases}$$

And the optimality conditions are determined from the equalities

$$(5) \qquad d_1 e^{-t} \alpha + v_x = 0, \quad \alpha(0,t) = \alpha(1,t) = 0.$$

The problem (1)–(3), (5), (6) was solved by the finite-difference scheme proposed in [3]. The numerical calculations were analysed and discussed.

To solve the inverse problem, we need to determine the functions (φ, α) from the additional measurements of $u(x,t)$ in computational domain. The sensitivity-based identifiability analysis was conducted.

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References

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