On isospectrality of genus three graphs

Elena Uharova
Novosibirsk State University, Novosibirsk, Russia
ukharova@gmail.com

Let G be a finite connected multigraph. Denote by $V(G)$ and $E(G)$ the set of vertices and edges of graph G, respectively.

For each $u, v \in V(G)$, we set a_{uv} to be equal to the number of edges between u and v. The matrix $A = A(G) = [a_{uv}], u, v \in V(G)$ is called the adjacency matrix of graph G.

Let $d(v)$ denote the valency of $v \in V(G)$, $d(v) = \sum_u a_{uv}$. $D = D(G)$ is a diagonal matrix indexed by $V(G)$, $d_{vv} = d(v), v \in V(G)$.

The matrix $L = L(G) = D(G) - A(G)$ is called the Laplacian matrix of G. We denote by $\mu(G, x)$ the characteristic polynomial of $L(G)$. For brevity's sake, we will call $\mu(G, x)$ the Laplacian polynomial of G.

Two graphs G and H are called isospectral if their Laplacian polynomials coincide: $\mu(G, x) = \mu(H, x)$.

Following [1] we denote the genus of graph G by $g = |E(G)| - |V(G)| + 1$ – the dimension of the first homology group of G.

A bridge is an edge of a graph G, whose deletion increases its number of connected components. A graph is called bridgeless if it doesn't contain any bridges.

The main result is the following theorem and hypothesis:

THEOREM. *Let G be a finite connected bridgeless genus three multigraph. Then G is isomorphic to the graph of one of eight types.*

HYPOTHESIS. *Two bridgeless genus three graphs belonging to the same type are isospectral if and only if they are isomorphic.*

P. Buser posed a similar hypothesis for Riemann surfaces. That problem is still open for Riemann surfaces, but was solved positively for genus two graphs in [2].

The hypothethis for genus three graphs was proved for two types.

References

[1] Baker M., Norine S., Harmonic morphisms and hyperelliptic graphs, *Int. Math. Res. Notes* **15** (2009) 2914–2955.

[2] Mednykh A., Mednykh I., Isospectral genus two graphs are isomorphic, *to appear in Discrete Mathematics* (2014).