On graphs with automorphism groups admitting a partition

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A finite group G is said to admit a *partition* if it can be expressed as a set-theoretic union of subgroups, with pairwise trivial intersections. Given a compact Riemann surface M with automorphism group G_0, we can obtain a quotient surfaces M/G_i, where G_i are subgroups of G_0. In [1] Accola derived the formula relating the genera of M/G_i to the orders of G_i provided G_0 admits a partition. Taniguchi, in [2], generalized this result to finite groups acting on a compact Hausdorff space.

Applying Bass-Serre theory of graph of groups and mentioned Taniguchi's result, for graphs with automorphism groups admitting a partition, we prove the statements analogous to ones in [1] by Accola. A graph here is a finite multigraph without loops. A graph is said to be γ-hyperelliptic if it is a two-fold covering of a genus γ graph. The background material on a covering of graphs one can find in [3].

Theorem 1. *Let X be a two-fold covering of a hyperelliptic graph Y of genus $g \geq 2$. Then X is γ-hyperelliptic for some $\gamma \leq \left[\frac{g-1}{2}\right]$.*

The immediate consequences of the theorem are the assertions below. The first one has been proved by I. A. Mednykh in [4] by sophisticated methods.

Corollary 1. *Suppose X is a graph of genus three which is a two-fold covering of a graph Y of genus two. Then X is hyperelliptic.*

Corollary 2. *If X is a graph of genus five which is a two-fold covering of a hyperelliptic graph of genus three, then X is hyperelliptic or 1-hyperelliptic.*

References

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