Volumes of polyhedra with symmetries

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We overview the volume calculations for polyhedra in Euclidean, spherical and hyperbolic spaces. We present some new results as well as open problems. The talk is mostly based on our joint work with Alexander Mednykh [1].

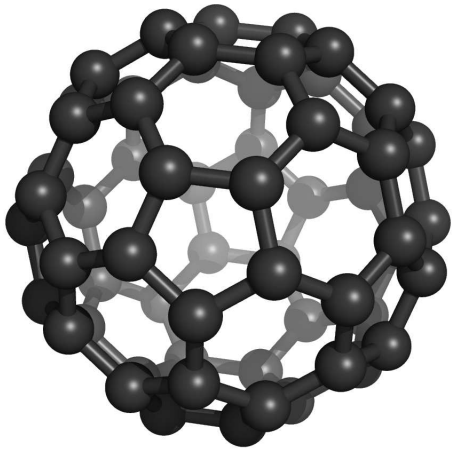
We prove the Sforza's formula for the volume of an arbitrary compact tetrahedron in H^3 and S^3.

In case of symmetrical tetrahedron the volume formula is essentially simplified. Firstly, this phenomenon was discovered by Lobachevsky who found the volume of ideal hyperbolic tetrahedron which is symmetrical by definition.

We present some results, which provide a solution for Seidel problem on the volume of ideal hyperbolic tetrahedron.

We obtain exact volume formulas for octahedra with symmetries in H^3 and S^3.

Consider a truncated icosahedron (so called fullerene C_{60} or soccer ball, pic 1) which is a semi-regular polyhedron and one of 13 Archimedean solids. It has $(2, 3, 5)$-symmetry which is very helpful in finding relations between its geometrical parameters. We solve an isoperimetrical problem for Euclidean C_{60} and find some relations between edge lengths, facet angles, dihedral angles, surface area and volume of C_{60} in H^3.



Pic 1. *Buckminsterfullerene or C_{60} is the smallest fullerene molecule*

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References

[1] N. Abrosimov, A. Mednykh, "Volumes of polytopes in constant curvature spaces", *Fields Institute Communications*, Vol. 70, 1–26 (2014). arXiv:1302.4919 [math.MG]