Polygons and polyhedron over finite field

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The objects of our research are regular polygons and regular polyhedra. The definitions of these notions can be found in [1, 2]. Note that over a finite field F_p one can define the polygon to be the set of straight lines passing through its vertices. The existence theorems of such polygons are given in [2]. The questions arising here are: in what finite field F_p can one realize the regular polyhedron and what will be the result of such a realization?

Our purpose is to find the characteristics p of the fields F_p over which there exists a certain regular polyhedron and to investigate the structure of such an object.

We define a tetrahedron over a finite field F_p to be the set of four triangles over F_p, intersecting pairwise along a common side. The tetrahedron over F_p is called regular if the squared distances between its vertices are all equal modulo p.

Theorem 1 *A regular tetrahedron exists over any finite field F_p.*

The similar results were obtained for a hexahedron and a octahedron.

Theorem 2 *A hexahedron exists over any finite field F_p.*

Theorem 3 *A octahedron exists over any finite field F_p.*

A dodecahedron and a icosahedron exist over F_p provided the number 5 is a square. More precisely, the following two theorems are valid.

Theorem 4 *A dodecahedron exists over the finite field F_p for $p \equiv \{0, 1, 4\} \pmod 5$.*

Theorem 5 *A icosahedron exists over the finite field F_p for $p \equiv \{0, 1, 4\} \pmod 5$.*

References

[1] Кокстер Г. С. М. *Введение в геометрию* // пер. с англ. под ред. Б.А. Розенфельда, И.М Яглома.- М.: Наука, Главная редакция физико-математической литературы, 1966, 648 с.

[2] Wildberger N. J. *Divine proportions: rational trigonometry to universal geometry* // Wild Egg Pty Ltd, Australia, 2005, P. 300.