On Groups whose Proper Schur Rings are Commutative

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A Schur ring $\mathscr{A}$ is called *Dedekind* if the formal sum of every $\mathscr{A}$-subgroup is in the center of $\mathscr{A}$. In this talk, we find all finite groups G such that every proper Schur ring over G is Dedekind.

Theorem 1. *Every proper Schur ring over G is Dedekind if and only if G is a Dedekind group or* $G \cong D_n$ *where* $n = 4$ *or* n *is a Fermat prime.*

As a consequence of this theorem, we find all finite groups G such that every proper Schur ring over G is commutative or symmetric, respectively.

Corollay 2. *Every proper Schur ring over G is commutative if and only if G is an abelian group,* $G \cong Q_8$ *or* $G \cong D_n$ *where* $n = 4$ *or* n *is a Fermat prime.*

Corollary 3. *Every proper Schur ring over G is symmetric if and only if G is an elementary abelian* 2*-group or* $G \cong C_n$ *where* $n = 4$ *or* n *is a Fermat prime.*

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