Deformations of polygonal dendrites in the plane

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Let $P\subset\mathbb{R}^2$ be a finite polygon homeomorphic to a disk, $V_P = \{A_1, ..., A_{n_P}\}$ be set of its vertices. We consider such a system of similarities $\mathcal{S} = \{S_1, \dots, S_m\}$ in $\mathbb{R}^2$ that:
(D1) for any $i \in I$ set $P_i = S_i(P)\subset P$;
(D2) for any $i \neq j$, $i, j \in I$, $P_i \cap P_j = V_{P_i} \cap V_{P_j}$;
(D3) $V_P\subset \bigcup\limits_{i\in I} S_i(V_P)$;
(D4) the set $\widetilde{P} = \bigcup\limits_{i=1}^{m} P_i$ is contractible.

The system $\mathcal{S}$, satisfying the conditions (**D1–D4**), is called a *contractible P-polygonal system of similarities.* As it was proved in [1], the attractor K of such system $\mathcal{S}$ is a dendrite.
If $\mathcal{S}$ satisfies **(D2–D4)** only, it is called a *generalised polygonal system.*

A generalised polygonal system $\mathcal{S}'$ is called δ-deformation of a P-polygonal system $\mathcal{S}$, if there is such homeomorphism $f : \mathbb{R}^2 \longrightarrow \mathbb{R}^2$, that $|f(x) - x| < \delta$ for any $x \in \mathbb{R}^2$ and $f(P_k) = P'_k$ for any $k = 1, ..., m$.

A vertex $A \subset V_P$ is called a *cyclic vertex*, if there is such multiindex $\mathbf{j} = j_1 j_2 \dots j_k$, that $S_{\mathbf{j}}(A) = A$. If $S_{\mathbf{j}}(z) = re^{i\varphi}(z - A) + A$, then the *parameter λ_A of the cyclic vertex A* is the number $\dfrac{\varphi}{\log r}$, where the choice of the value of φ is defined by the geometric configuration of the system $\mathcal{S}$.

A point $B \in \cup_{i=1}^{m} V_{P_i}$ is *subordinate* to a cyclic vertex A, if for some multiindex $\mathbf{j}, S_{\mathbf{j}}(A) = B$.

Parameter matching condition. *We say that a system $\mathcal{S}$ satisfies the parameter matching condition if for any $B \in \cup_{i=1}^{m} V_{P_i}$ and cyclic vertices A, A' such B is subordinate to both A and A', $\lambda_A = \lambda_{A'}$.*

If the parameter matching condition for the generalised polygonal system eS is violated at some point $B \in \cup_{i=1}^{m} V_{P_i}$, then its attractor K is not simply-connected near the point B and therefore is not a dendrite.

Theorem. *For any contractible P-polygonal systen $\mathcal{S}$ there is such $\delta > 0$, that for any δ-deformation $\mathcal{S}'$ of the system $\mathcal{S}$, satisfying the parameter matching condition, the attractor $K(\mathcal{S}')$ is a dendrite, homeomorphic to $K(\mathcal{S})$.*

References

[1] M. Samuel, A. V. Tetenov, D. A. Vaulin, Self-Similar Dendrites Generated by Polygonal Systems in the Plane. *Sib. Electron. Math. Rep.* **14** (2017) 737–751.