Graphs and metrics: the partition case

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Given an ordered partition $\Pi = \{P_1, P_2, ..., P_t\}$ of the vertices of a connected graph G, the *partition representation* of a vertex $v \in V$ with respect to the partition Π is the vector $r(v|\Pi) = (d(v, P_1), d(v, P_2), ..., d(v, P_t))$, where $d(v, P_i)$, with $1 \leq i \leq t$, represents the distance between the vertex v and the set P_i, that is $d(v, P_i) = \min_{u \in P_i}\{d(v, u)\}$. The partition Π is a *resolving partition* of G if for every pair of distinct vertices $u, v \in V$, $r(u|\Pi) \neq r(v|\Pi)$ (in such case, we also say that there is a set $P_i \in \Pi$ which resolves the pair of vertices u, v). The *partition dimension* of G is the minimum number of sets in any resolving partition of G and is denoted by $pd(G)$. Concepts above were presented first in [1].

On the other hand, a set W of vertices of G *strongly resolves* two different vertices $x, y \notin W$, if either $d_G(x, W) = d_G(x, y) + d_G(y, W)$ or $d_G(y, W) = d_G(y, x) + d_G(x, W)$. An ordered vertex partition $\Pi = \{U_1, U_2, ..., U_k\}$ of a graph G is a *strong resolving partition* for G if every two different vertices of G belonging to the same set of the partition are strongly resolved by some set of Π. A strong resolving partition of minimum cardinality is called a *strong partition basis* and its cardinality the *strong partition dimension*, which is denoted by $pd_s(G)$. Concepts above were presented first in [2].

Now, the Cartesian product of two graphs $G = (V_1, E_1)$ and $H = (V_2, E_2)$ is the graph $G \square H = (V, E)$, such that $V = \{(a, b) \ : \ a \in V_1, \ b \in V_2\}$ and two vertices $(a, b), (c, d) \in V$ are adjacent in $G \square H$ if and only if, either

- $a = c$ and $bd \in E_2$, or
- $b = d$ and $ac \in E_1$.

Also,the strong product of G and H is the graph $G \boxtimes H = (V, E)$, such that $V = \{(a, b) \ : \ a \in V_1, \ b \in V_2\}$ and two vertices $(a, b), (c, d) \in V$ are adjacent in $G \boxtimes H$ if and only if, either

- $a = c$ and $bd \in E_2$, or
- $b = d$ and $ac \in E_1$, or
- $ac \in E_1$ and $bd \in E_2$.

For more information on product graphs we recommend the book [3]. Several results concerning the (strong) partition dimension of graphs with some emphasis in product graphs are presented in this work.

References

[1] G. Chartrand, E. Salehi, P. Zhang, The partition dimension of a graph. *Aequationes Mathematicae* (1-2) **59** (2000) 45–54.

[2] I. González Yero, On the strong partition dimension of graphs. *The Electronic Journal of Combinatorics* **21**(3) (2014) P3.14.

[3] R. Hammack, W. Imrich, S. Klavžar, *Handbook of product graphs*, 2nd Edition. Discrete Mathematics and its Applications. CRC Press (2011).