Integral Cayley graphs over finite groups

Daria Lytkina
Siberian State University of Telecommunications and Information Sciences, Novosibirsk, Russia
daria.lytkin(at)gmail.com

This is joint work with Elena Konstantinova

Suppose that S is a nonempty subset of a finite group G, containing with every element its inverse, i. e. $S = S^{-1} = \{s^{-1} \mid s \in S\}$. The *Cayley graph* $\Gamma = Cay(G, S)$ of a group G associated with S is an undirected graph with the vertex set identified with G, and vertices $g, h \in G$ are joined by an edge if and only if there exists $s \in S$ such that $s = g^{-1}h$. A graph Γ is said to be *integral*, if all eigenvalues of its adjacency matrix are integers [1].

Integral Cayley graphs over abelian, dihedral and cyclic groups were investigated in [2–4].

In this talk we present new results on integral Cayley graphs over finite groups.

Theorem 1. *Let G be a finite nilpotent group and $S = S^{-1}$ be a nonempty subset of G. If S is normal, i. e. $S^G = S$, and with every element $s \in S$ it contains also all generators of the cyclic group $\langle s \rangle$, then $\Gamma = Cay(G, S)$ is integral.*

Corollary. *A Cayley graph $\Gamma = Cay(G, S)$ of a 2-group G generated by a normal set S of involutions is integral.*

Theorem 2. *Let $G = S_n$ be the symmetric group of degree $n \geqslant 2$ and S be the set of all transpositions of G. Then the graph $\Gamma = Cay(G, S)$ is integral.*

Theorem 3. *Let $G = A_n$ be the alternating group of degree $n \geqslant 2$ and $S = \{(1ij) \mid 2 \leqslant i, j \leqslant n, i \neq j\}$. Then the graph $\Gamma = Cay(G, S)$ is integral. Its spectrum coincides with the set*

$$\{-n + 1, 1 - n + 1, 2^2 - n + 1, \ldots, (n - 1)^2 - n + 1\}.$$

Acknowledgments. The work has been supported by RFBR Grant 17-51-560008.

References

[1] F. Harary, A. J. Schwenk, Which graphs have integral spectra? *Graphs and Combinatorics* **390** (1974) 45–51.

[2] W. Klotz, T. Sander, Integral Cayley graphs over abelian groups. *Electr. J. Combin.*, **7** (2010) 13 pp.

[3] Lu Lu, Qiongxiang Huang, Xueyi Huang, Integral Cayley graphs over dihedral groups. *J. Algebr. Comb.* (2017) (https://doi.org/10.1007/s10801-017-0787-x).

[4] W. So, Integral circulant graphs, *Discrete mathematics.* **306** (2005) 153–158.