

Arithmetics and combinatorics of circulant graphs

Alexander Mednykh

Sobolev Institute of Mathematics, Novosibirsk, Russia

Novosibirsk State University, Novosibirsk, Russia

smedn(at)mail.ru

We study arithmetical and combinatorial properties of a few infinite families of circulant graphs. Among them are circulant graphs with constant jumps, circulant graphs with odd valency of vertices and circulant graphs with unbounded jumps. We investigate the Smith normal form of Laplacian for these graphs and determine the structure of the corresponding critical group [1]. The obtained results are applied to find the number of spanning trees for the above mentioned families of graphs [2].

Exact analytical formulas for the number of spanning trees in terms of Chebyshev polynomials are derived, and their asymptotics is found. As a consequence, we show that the thermodynamic limit of a family of circulant graphs coincides with the small Mahler measure of the associated Laurent polynomial. Also, we investigate pure arithmetical properties for the number of spanning trees of circulant graphs.

Acknowledgments. The work has been supported by RFBR (grants 16-31-00138, 18-01-00420 and 18-501-51021) and by the program of fundamental researches of the SB RAS no.I.1.2., project 0314-2016-0007.

References

- [1] A. D. Mednykh, I. A. Mednykh, On the structure of the Jacobian group for circulant graphs. *Doklady Mathematics* **94**(1) (2016) 445–449.
- [2] A. D. Mednykh, I. A. Mednykh, Asymptotics and arithmetical properties of complexity for circulant graphs. *Doklady Mathematics* **97**(2) (2018) 147–151.