On eigenfunctions of Hamming graphs with minimum support

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Denote by $H(n,q)$ the Hamming graph with parameters n and q. It is well-known that the set of eigenvalues of the adjacency matrix of $H(n,q)$ is $\{\lambda_i = n(q-1) - qi \mid i = 0,1,\dots,n\}$. Denote by $U_j(n,q)$ an eigenspace corresponding to λ_j. The space $U_i(n,q) + \ldots + U_j(n,q)$ for $i \leq j$ is denoted by $U_{[i,j]}(n,q)$.

In this work we investigate some extremal properties of eigenspaces of the Hamming graph. We consider the problem of finding the minimum cardinality of the support of eigenfunctions of the Hamming graphs $H(n,q)$. This problem is directly related to the problem of finding the minimum possible difference of two combinatorial objects. In more details, these connections are described in [3], where the minimum cardinality of the support of an eigenfunction of the Grassmann graph with the smallest eigenvalue was found. The problem of finding the minimum size of the support of eigenfunctions was studied for the Johnson graphs in [7], for the Doob graph in [1], for the cubic distance-regular graph in [5] and for the Paley graphs in [2]. For the Hamming graph this problem was investigated in [4, 6]. In this paper we obtain the more general result for the functions from the sum of eigenspaces of the Hamming graph. We prove the following result:

Theorem 1. *Let $f : H(n,q) \longrightarrow \mathbb{R}$, $f \in U_{[i,j]}(n,q)$ and $f \not\equiv 0$. Then the following statements are true:*

1. $|f| \geq 2^i(q-1)^i q^{n-i-j}$ *for* $n \geq i+j$ *and* $q \geq 3$.
2. $|f| \geq 2^i(q-1)^{n-j}$ *for* $i+j > n$ *and* $q \geq 4$.

Moreover, we give a characterization of functions with the minimum cardinality of the support for the first case of theorem and for the second case for $i = j$ and $q > 4$.

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