Weakly distance-regular digraphs

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This is a joint work with Akihiro Munemasa and Kaishun Wang

The concept of weakly distance-regular digraphs was firstly introduced by Suzuki and Wang in [2]. Such digraphs have a close relation with association schemes. Some special families of weakly distance-regular digraphs were classified. See [1, 2] for valency 2, [3–5] for valency 3, [1] for thin case and [6] for quasi-thin case.

In this talk, we firstly consider a special family of pseudocyclic association schemes, and obtain the following result.

Theorem 1. Let $(X, \{R\}_{i=0}^{d})$ be an commutative pseudocyclic association scheme generated by a non-symmetric relation R_1 with $p_{1,1}^{1^*} \neq 0$. If

$$\{R_1, R_{1^*}, R_2\} \supseteq R_1^2 \text{ and } \{R_0, R_1, R_{1^*}, R_2, R_{2^*}\} \supseteq R_1 R_{1^*}$$

with $2 \neq 2^*$, then one of the following holds:

(i) $d = 2$ and $|X| = 3$.

(ii) $d = 2$ and $|X| \equiv 3 \pmod 4$.

(iii) $d = 4$ and $|X| = 13$.

As a by-product of Theorem 1, we determine all the primitive weakly distance-regular circulant digraphs in the following result.

Theorem 2. *If Γ is a primitve weakly distance-regular circulant digraph, then Γ is isomorphic to one of the following digraphs:*

(i) *the circuit of length p, where p is prime.*

(ii) *the Paley digraph of p vertices, where $p \equiv 3 \pmod 4$ is prime.*

(iii) $\mathrm{Cay}(\mathbb{Z}_{13}, \{1, 3, 9\})$.

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