Applications of semidefinite programming, symmetry, and algebra to graph partitioning problems

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This is joint work with Renata Sotirov

We will present semidefinite programming (SDP) and eigenvalue bounds for several graph partitioning problems.

The graph partition problem (GPP) is about partitioning the vertex set of a graph into a given number of sets of given sizes such that the total weight of edges joining different sets — the cut — is optimized. We show how to simplify known SDP relaxations for the GPP for graphs with symmetry so that they can be solved fast, using coherent algebras.

We then consider several SDP relaxations for the max-k-cut problem, which is about partitioning the vertex set into k sets (of arbitrary sizes) such that the cut is maximized. For the solution of the weakest SDP relaxation, we use an algebra built from the Laplacian eigenvalue decomposition — the Laplacian algebra — to obtain a closed form expression that includes the largest Laplacian eigenvalue of the graph. This bound is exploited to derive an eigenvalue bound for the chromatic number of a graph. For regular graphs, the new bound on the chromatic number is the same as the well-known Hoffman bound. We demonstrate the quality of the presented bounds for several families of graphs, such as walk-regular graphs, strongly regular graphs, and graphs from the Hamming association scheme.

If time permits, we will also consider the bandwidth problem for graphs. Using symmetry, SDP, and by relating it to the min-cut problem, we obtain best known bounds for the bandwidth of Hamming, Johnson, and Kneser graphs up to 216 vertices.