On 3-generated lattices with special elements among generators

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Definitions

- We call an element s of a lattice L standard, if the equality
$$x \wedge (y \vee s) = (x \wedge y) \vee (x \wedge s)$$
holds for any elements x and y of L.

- We call an element s of a lattice L dual standard, if the equality
$$x \vee (y \wedge s) = (x \vee y) \wedge (x \vee s)$$
holds for any elements x and y of L.

Theorem(G.Grätzer, E.T.Shmidt, 1961). Lattice, which is generated by three elements, two of which are standard, is distributive and contains at most 18 elements.

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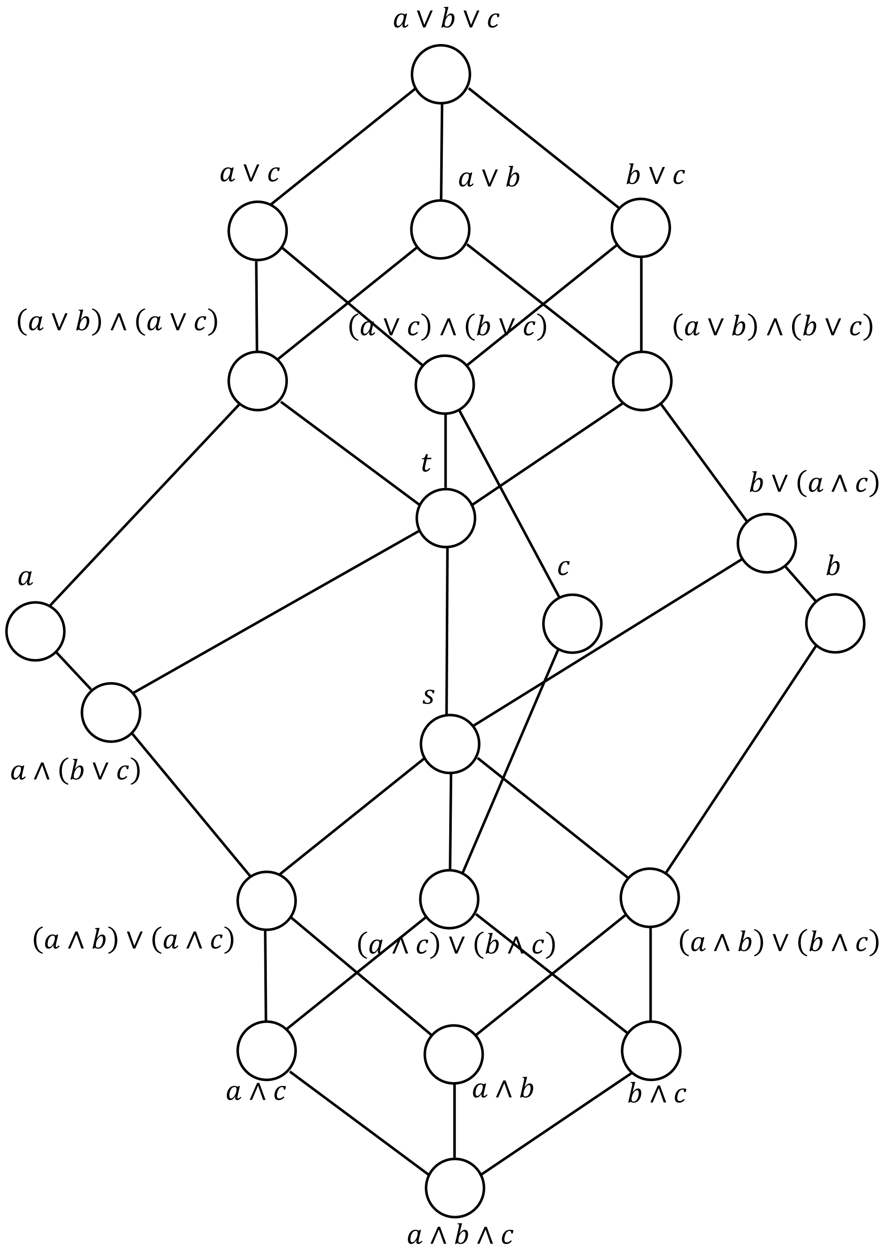
Theorem(N.A.Minigulov, 2016)

Let L be a 3-generated lattice. If one of generators of L is standard and another generator is dual standard then L contains at most 21 elements and L is a homomorphic image of the lattice presented on figure.



Definitions

- We call an element n of a lattice L neutral, if the equality

$$(n \vee x) \wedge (n \vee y) \wedge (x \vee y) = (n \wedge x) \vee (n \wedge y) \vee (x \wedge y)$$

holds for any elements x and y of L.

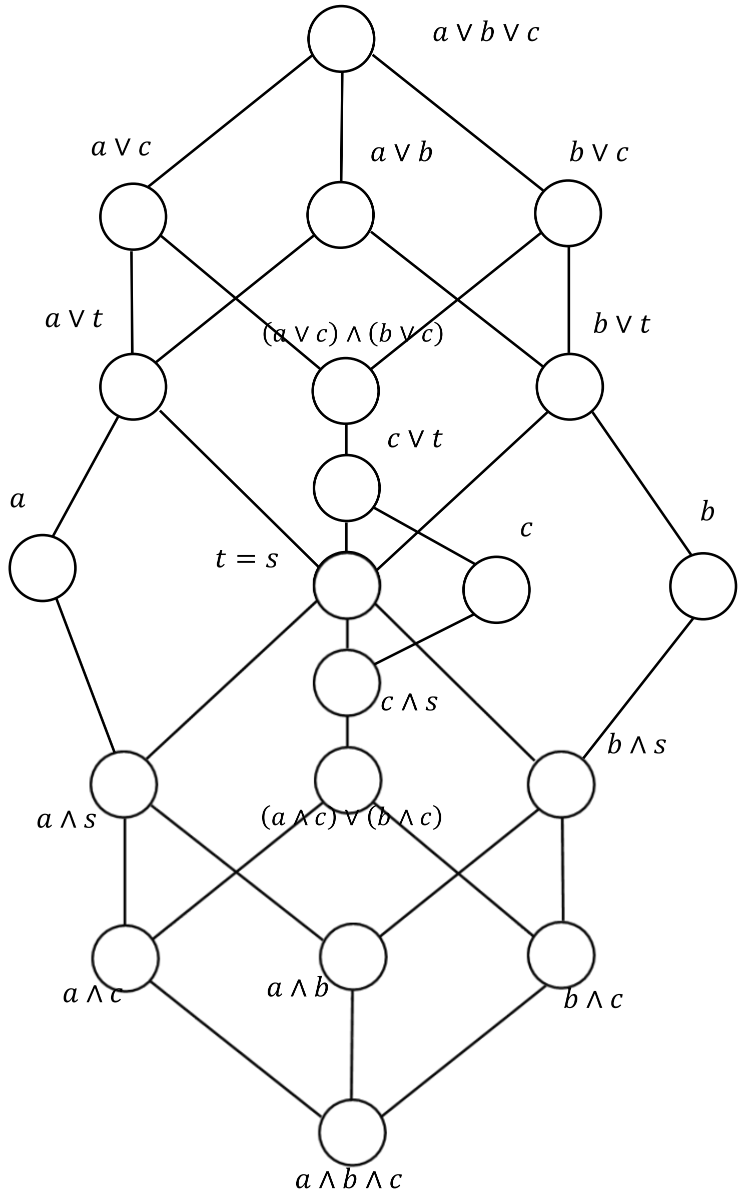
- We call a pair of elements (a, b) of a lattice L neutral, if the equality

$$(a \vee x) \wedge (b \vee x) \wedge (a \vee b) = (a \wedge x) \vee (b \wedge x) \vee (a \wedge b)$$

holds for any element x of L.

Theorem(A.G.Gein, N.A.Minigulov, 2016)

Let L be a 3-generated lattice. If two of generators of L are neutral pair then L contains at most 20 elements and L is a homomorphic image of the lattice presented on figure.



Thank you for attention!