Classification of genus 1 virtual knots of low complexity

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Introduction

- Knots in thickened surfaces such as $S \times I$
- Projections and diagrams of such knots are analogous to the classical ones

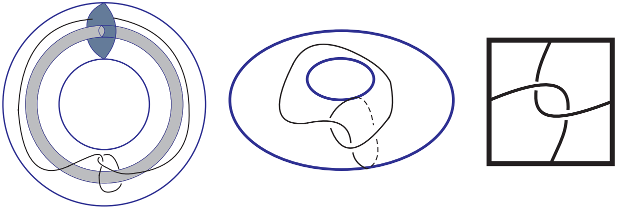
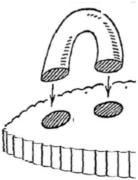
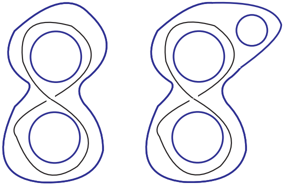


Figure : An example: a knot in $T \times I$

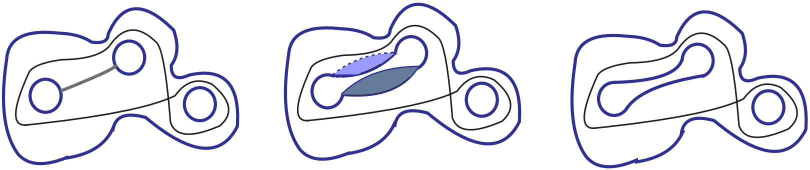
Introduction

- Stabilization





- Destabilization



Introduction

Definition

Virtual knot is the equivalence class of the knots in thickened surfaces modulo

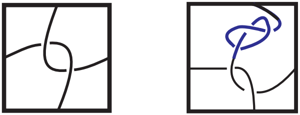
- *homeomorphisms such as* $h : (S \times I, K) \to (S' \times I, K')$
- *stabilizations (destabilizations)*

- Virtual knot theory was introduced by L. Kauffman in 1996.

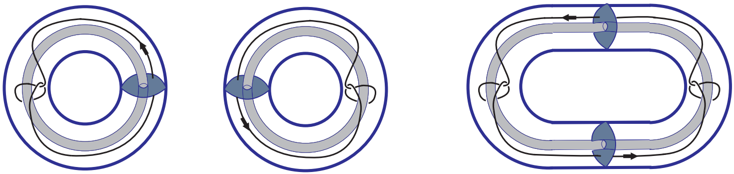
Definition

A genus of knot K *is the minimal genus of a surface* S *such that* K *is situated in* $S \times I$

- We consider prime (that is non composite) knots in $T \times I$ such that they do not allow destabilizations
- Composite knot K:
  - K is a connected sum of a knot in $T \times I$ and a nontrivial knot in S^3



  - K is a nontrivial circular connected sum of knots in $T \times I$



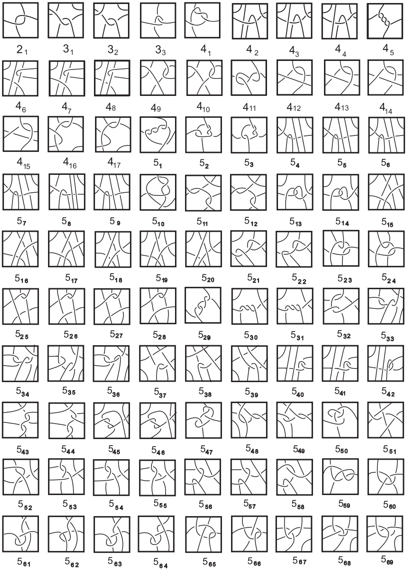
- Prime projection and diagram

Theorem

There exist exactly 90 different genus 1 prime virtual knots with at most 5 crossings

Proof:

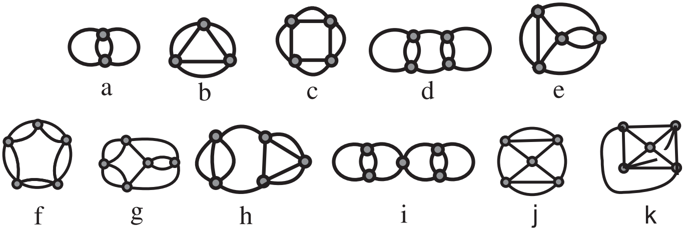
- Enumeration of graphs
- Enumeration of projections in T
- Recovering of diagrams in T
- All these diagrams are distinct



Enumeration of graphs

Theorem

There exist exactly 11 abstract regular graphs of degree 4 having at most 5 vertices and no loops.



Only graph k has no loop and multiple edge
All others graphs are obtained from a circle using following operations:

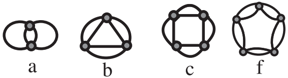
- Apply a loop
- Apply a multiple edge

Enumeration of projections in T

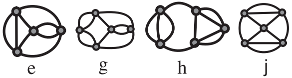
Theorem

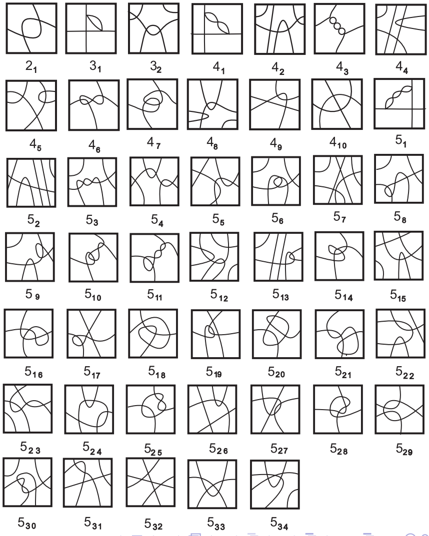
There exist exactly 47 different prime projections in T with at most 5 crossings

- Cyclically ordered chain of n circles



- Double edge

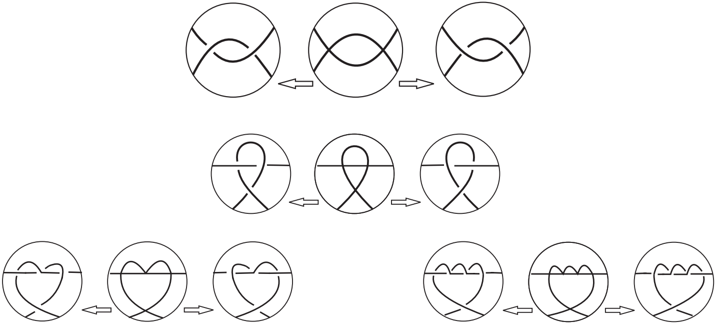




Recovering of diagrams in T

Ideas:

- We can arbitrarily choose a type of one crossing
- There are only two ways to indicate the under- and over-crossings for the fragment shown in the center:



All these diagrams are distinct

Generalized Kauffman polynomial:

$$X(K) = (-a)^{-3w(K)} \sum_{s} a^{\alpha(s)-\beta(s)} (-a^2 - a^{-2})^{\gamma(s)} x^{\delta(s)}$$

- $\alpha(s)$, $\beta(s)$ are the numbers of markers A and B in a given state s
- $\gamma(s)$, $\delta(s)$ - are the numbers of trivial and nontrivial circles in T in a given state s
- $w(K)$ is the writhe of the diagram
- The sum is taking over all states

Figure : Markers

Figure : Trivial and nontrivial circle in T

Virtual knots diagrams in the plane

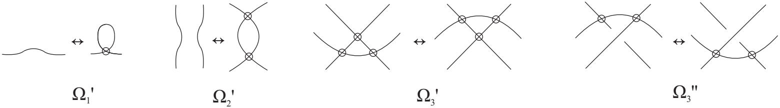
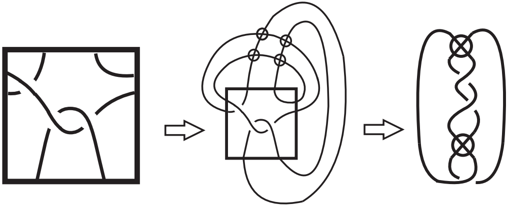


Figure : Virtual and semi-virtual versions of Reidemeister moves

- we close knot diagrams in the plane by analogy with the braid closure. The obtained arcs intersect by the virtual crossings.
- we apply a sequence of $\Omega_1', \Omega_2', \Omega_3'$ Ω_3'' for removing some virtual crossings



Virtual knots diagrams in the plane