# Some classes of fibered links

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# Fibered links, grid diagrams

**Definition**

*An oriented link in $S^3$ is called fibered if its exterior may be represented as a fiber bundle of a family of Seifert surfaces over $S^1$.*

**Definition**

*A planar diagram of a link is called a grid diagram if:*

- *It is partially linear and contains only vertical and horizontal edges*
- *The edges are in general position*
- *In each self-crossing the vertical edge passes over the horizontal edge*

*For the case of oriented links all the vertices are painted black or white in such a manner that the vertical edges are oriented from black vertex to white.*

# Diagonal links

**Definition**

*Let $\sigma \in S_n$ be a permutation without fixed points. Then $\mathrm{diag}(\sigma)$-type link is defined by its grid diagram with the following sets of vertices:*

- *Black vertices:* $\{(i,i) | i = 1, \ldots, n\}$.
- *White vertices:* $\{(i,\sigma(i)) | i = 1, \ldots, n\}$.

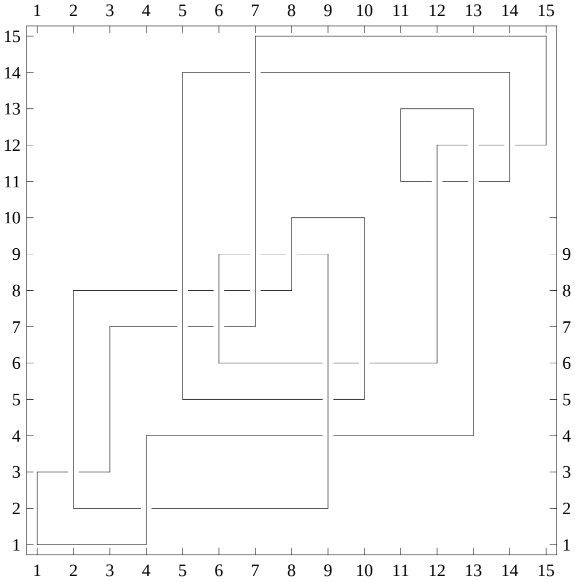
**Definition**

*A permutation $\sigma \in S_n$ is called non-separable if for each pair $i < j$, $(i,j) \neq (1,n)$ we have $\sigma(\{i,\ldots,j\}) \neq \{i,\ldots,j\}$*

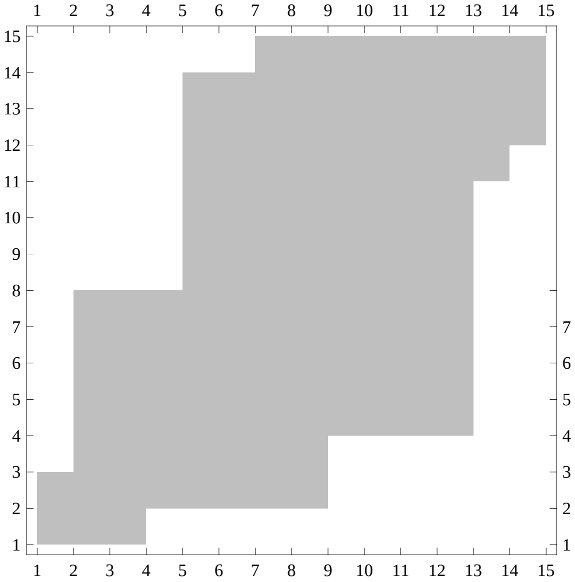
**Theorem**

*A $\mathrm{diag}(\sigma)$-type link is fibered iff $\sigma$ is non-separable.*

# Knot $\mathrm{diag}(3, 8, 7, 1, 14, 9, 15, 10, 2, 5, 13, 6, 4, 11, 12)$

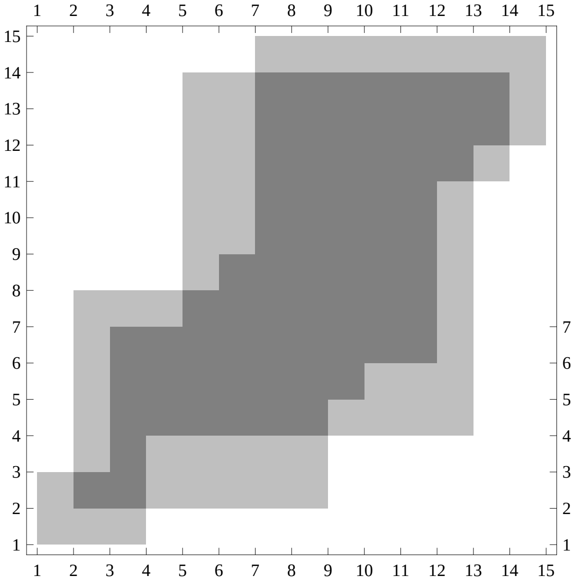


# Level 1



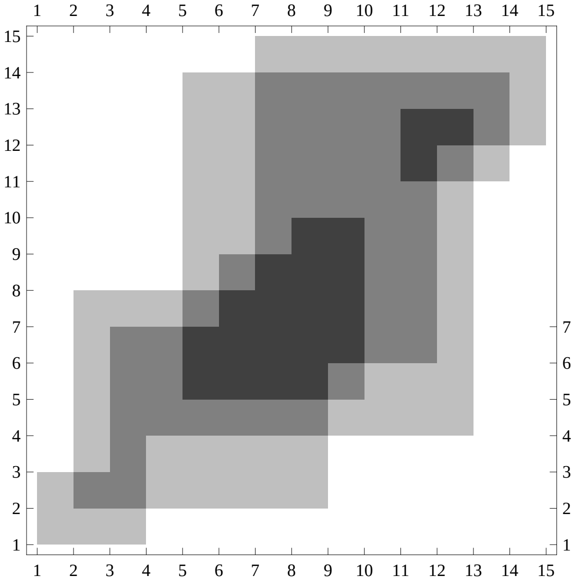
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

# Level 2



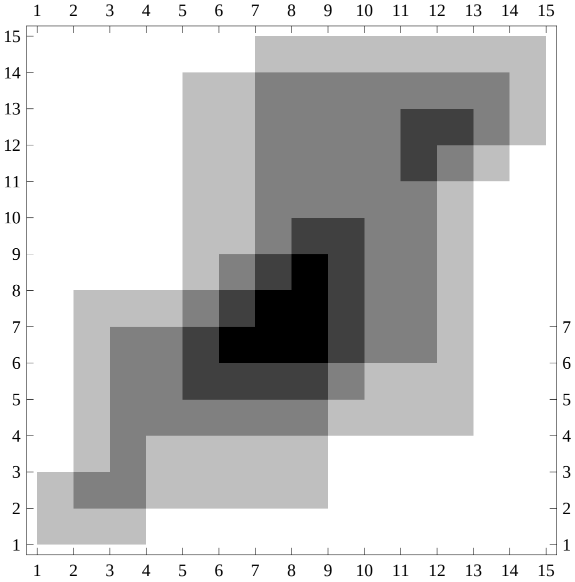
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

# Level 3

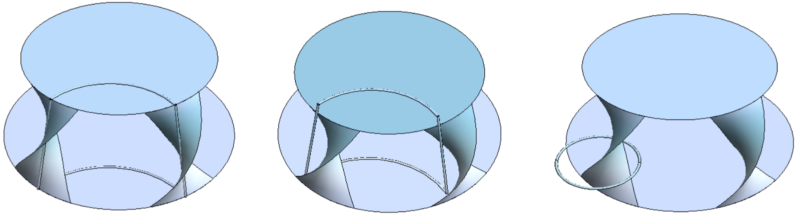


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

# Level 4


1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
15 14 13 12 11 10 9 8 7 6 5 4 3 2 1
7 6 5 4 3 2 1
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15

# The push-off map



The push-off map $f_* : \pi_1(M) \to \pi_1(S^3 \setminus M)$ is surjective.
Hence our link is fibered:
John Stallings. On fibering certain 3-manifolds. Topology of 3-manifolds, 95-109. Prentice-Hall. New Jersey (1962).

# $(n, p, q)$-periodic knots

**Definition**

*Let* $n, p, q \in \mathbb{N}$ *with* $(n, p) = (n, q) = 1, 1 \leq p, q \leq n - 1$. *Then* $(n, p, q)$*-periodic knot is defined by its grid diagram with the following sets of vertices:*

- *Black vertices:* $\{(1 + iq, 1 + ip) | i = 0, \ldots, n - 1\}$.
- *White vertices:* $\{(1 + iq, 1 + (i + 1)p) | i = 0, \ldots, n - 1\}$.

**Theorem**

*All* $(n, p, q)$*-periodic knots are fibered.*

# Heegard Floer Homology

Peter Ozsvath and Zoltan Szabo. Holomorphic disks and knot invariants. Adv. Math., 186(1):58–116, 2004

$$\widehat{\mathrm{HFK}}(K) = \bigoplus_{m,s} \widehat{\mathrm{HFK}}_m(K,s)$$

Euler characteristic:

$$\Delta_K(t) = \sum_{m,s} (-1)^m \mathrm{rank}\, \widehat{\mathrm{HFK}}_m(K,s) \cdot T^s$$

The differential preserves Alexander grading.
We use combinatoric description of $\widehat{\mathrm{HFK}}(K)$ from the article of Ciprian Manolescu: http://www.math.ucla.edu/~cm/proc_ecm.pdf

# Theorems about genus and fiberedness

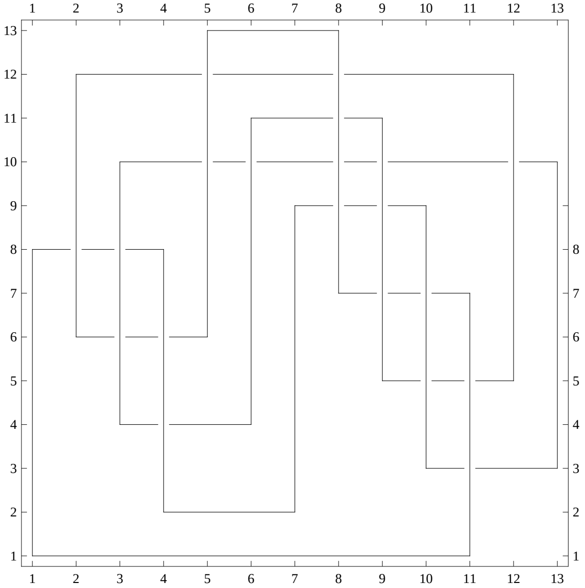
Peter Ozsvath and Zoltan Szabo. Holomorphic disks and genus bounds. Geometry and Topology, 8(2004), 311–334.

$$g(K) = \max \{s \in \mathbb{Z} | \widehat{\mathrm{HFK}}_*(K, s) \neq 0\}$$

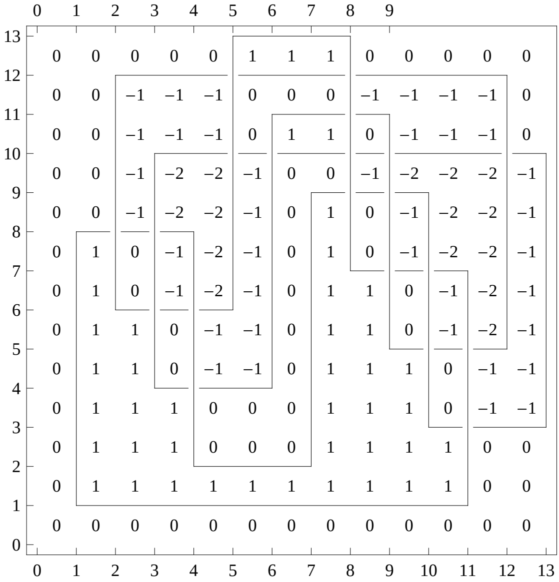
Y. Ni. Knot Floer homology detects fibered knots. Inventiones Mathematicae, 177(2009), no. 1, 235-238.

$$\dim \widehat{\mathrm{HFK}}_*(K, g(K)) = 1$$

# $(13, 7, 3)$-periodic knot



# Matrix $a$ for the diagram of $(13, 7, 3)$-periodic knot



| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | −1 | −1 | −1 | 0 | 0 | 0 | −1 | −1 | −1 | −1 | 0 |
| 0 | 0 | −1 | −1 | −1 | 0 | 1 | 1 | 0 | −1 | −1 | −1 | 0 |
| 0 | 0 | −1 | −2 | −2 | −1 | 0 | 0 | −1 | −2 | −2 | −2 | −1 |
| 0 | 0 | −1 | −2 | −2 | −1 | 0 | 1 | 0 | −1 | −2 | −2 | −1 |
| 0 | 1 | 0 | −1 | −2 | −1 | 0 | 1 | 0 | −1 | −2 | −2 | −1 |
| 0 | 1 | 0 | −1 | −2 | −1 | 0 | 1 | 1 | 0 | −1 | −2 | −1 |
| 0 | 1 | 1 | 0 | −1 | −1 | 0 | 1 | 1 | 0 | −1 | −2 | −1 |
| 0 | 1 | 1 | 0 | −1 | −1 | 0 | 1 | 1 | 1 | 0 | −1 | −1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 0 | −1 | −1 |
| 0 | 1 | 1 | 1 | 0 | 0 | 0 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

# $\bar{a}$ - the average of matrix $a$ over the $\mathbb{Z}_n$-action