Abstracts of the International Conference

“KNOTS, BRAIDS AND AUTOMORPHISM GROUPS”

Sobolev Institute of Mathematics, Novosibirsk

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Contents

SOME PROPERTIES OF VIRTUAL, FLAT VIRTUAL AND GAUSS BRAID GROUPS

VALERIY BARDAKOV

In the paper [1] for every closed virtual braid was defined a group and proved that this group is an invariant of the corresponding virtual link. To do this we used a representation $VB_n \to \mathrm{Aut}(F_{n+1})$ of the virtual braid group VB_n into the group of automorphisms of free group F_{n+1} of rank $n+1$.

In the present report we discussed some properties of the flat virtual braid group FB_n, Gauss braid group GB_n and construct some representations $FB_n \to \mathrm{Aut}(\widetilde{F}_{n+2})$ into the automorphism group of some quotient $\widetilde{F}_{n+2}$ of the free group F_{n+2}. The similar representation will construct for GB_n. Using these representations we define groups of flat virtual links and Gauss links.

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Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail address: bardakov@math.nsc.ru

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RESIDUAL PROPERTIES AND LINEAR REPRESENTATION OF GROUPS

OLEG BRYUKHANOV

Let $\mathfrak{X}$ be a class of groups. A group G is referred to as super-residually $\mathfrak{X}$ if for every finite subset $X \subset G$ there exists normal subgroup $N \lhd G$ such that $G/N \in \mathfrak{X}$ and $xN \neq yN$ for all $x, y \in X$. A group G is referred to as residually $\mathfrak{X}$ if for every element $x \in G$ there exists normal subgroup $N \lhd G$ such that $G/N \in \mathfrak{X}$ and $xN \neq N$.

In the paper [1] some sufficient conditions for an isomorphic representation over a field of a group by matrices and a criterion of the linear representation for finitely generated groups are presented. The criterion is based by fact that a group must be super-residually $\mathcal{L}(n, R)$ where $\mathcal{L}(n, R)$ is a class of $n \times n$-matrix groups with coefficients from some class of commutative associative rings involving all fields. This result generalizes a similar criterion due to Mal'cev [2].

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Siberian University of Consumer Cooperatives, Novosibirsk, Russia
E-mail address: bryuoleg@ngs.ru

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PALINDROMIC WIDTH OF FINITELY GENERATED GROUPS

KRISHNENDU GONGOPADHYAY

Let G be a group with a set of generators X. A reduced word in the alphabet $X^{\pm 1}$ is a palindrome if it reads the same forwards and backwards. The palindromic length $\ell_P(g)$ of an element g in G is the minimum number k such that g can be expressed as a product of k palindromes. The palindromic width of G with respect to X is defined to be the supremum of the set of palindromic lengths in (G, X).

In this presentation, we shall discuss recent results on the palindromic width of finitely generated groups. We shall show an estimate of palindromic width of finitely generated free nilpotent groups. For arbitrary solvable groups of step at most 3, it will be shown that if G is a finitely generated solvable group that is an extension of an abelian group by a group satisfying the maximal condition for normal subgroups, then the palindromic width of G is finite. For solvable groups of step 3, we have a complete answer: every finitely generated 3-step solvable group has finite palindromic width with respect to any finite generating set. Palindromic widths of metabelian groups will also be discussed. The talk is based on my joint work with Valeriy Bardakov.

Indian Institute of Science Education and Research, Mohali, India
E-mail address: krishnendug@gmail.com

REPRESENTATIONS OF THE VIRTUAL BRAID GROUPS TO THE ROOK ALGEBRAS

KONSTANTIN GOTIN

Study of braid groups and their generalizations occupies an important place in the modern three-dimensional topology. This is a useful approach to construct knots and links invariants. It is known that the virtual braid group VB_n comparing to the classical braid group B_n has additional generators as well as additional relations (see, for example [1]).

In [2] it was constructed a representation of the group B_n in the group of invertible elements of the subalgebra $\mathbb{C}P_n$ of the rook algebra $\mathbb{C}R_n$.

We will demonstrate that to extend the braid group representation given in [2] to a virtual braid group representation one will need to extend the algebra $\mathbb{C}P_n$ in some sense. We will construct a representation of the group VB_n to rook algebra $\mathbb{C}R_n$ such that its restriction on B_n coincides with the representation given in [2].

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Novosibirsk State University, Novosibirsk, 630090, Russia
E-mail address: gktin@yandex.ru

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STRONG REALITY AND TOTAL ORTHOGONALITY OF SPECIAL 2-GROUPS

DILPREET KAUR

A group G is said to be real if every element of G belongs to conjugacy class of its inverse. Strongly real groups and totally orthogonal groups are two important subclasses of real groups. We give examples of groups which are in one subclass but not the other. All known such examples lie in the class of special 2-groups. The theory of quadratic forms over the field of characteristic 2 is used to study the properties of special 2-groups.

Indian Institute of Science Education and Research, Mohali, India
E-mail address: dilpreetmaths@gmail.com

FINITE p-GROUPS WITH MAXIMUM NUMBER OF CLASS-PRESERVING AUTOMORPHISMS

MANOJ KUMAR YADAV

Motivated by a result of Burnside (proved a century ago), I studied finite p-groups having maximum number of (conjugacy) class-preserving automorphisms. I'll discuss about this work, where I show that in most of the cases, finite p-groups having maximum number of class-preserving automorphisms actually have minimum number of such automorphisms which are non-obvious. Obvious class-preserving automorphisms are inner automorphisms.

Harish-Chandra Research Institute, Allahabad, India
E-mail address: myadav@hri.res.in

ON SIMPLICIAL RESOLUTIONS OF FRAMED LINKS

FENGLING LI

In this talk, we investigate the simplicial groups obtained from the link groups of naive cablings on any given framed link. The main result states that the resulting simplicial groups have the homotopy type of the loop space of a wedge of 3-spheres. This gives simplicial group models for some loop spaces using link groups. This is a joint work with Fengchun Lei and Jie Wu.

Dalian University of Technology, Dalian, China
E-mail address: dutlfl@163.com

BRUNNIAN BRAIDS AND LIE ALGEBRAS

JINGYAN LI

Brunnian braids play an important roll in homotopy groups of spheres. In this article, we study the graded Lie algebra of the lower central series associated with Brunnian subgroup of the pure braid group. A simple presentation of this Lie algebra is obtained. This is a joint work with V. V. Vershinin and J. Wu.

Shijiazhuang Tiedao University, Shijiazhuang, China
E-mail address: yanjinglee@163.com

ON DISCRETENESS OF 2-GENERATOR SUBGROUPS OF $\mathrm{PSL}(2,\mathbb{C})$

ALEXANDER MASLEY

It is well known that the group $\mathrm{PSL}(2,\mathbb{C})$ acts on the hyperbolic 3-space $\mathbb{H}^3$ as the group of all orientation preserving isometries. Every element in $\mathrm{PSL}(2,\mathbb{C})$ is elliptic, parabolic, or loxodromic. It was shown by Jørgensen [1], that a nonelementary group $G < \mathrm{PSL}(2,\mathbb{C})$ is discrete if and only if each 2-generator subgroup of G is discrete.

We will represent a sufficient discreteness conditions for groups with two nonparabolic generators obtained in [2, 3]. Using these results we will give a partial answer to the Maskit's question [4] about the discreteness of some family of 2-generator groups.

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Novosibirsk State University, Novosibirsk, Russia
E-mail address: masley.alexander@gmail.com

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LOCAL REPRESENTATIONS OF BRAID GROUP AND ITS GENERALIZATIONS

YULIYA A. MIKHALCHISHINA

The linear representations of braid group B_n as well as those of virtual braid group VB_n, welded braid group WB_n and singular braid group SB_n are under investigation.

The local linear representations as well as the local homogeneous representations of braid group B_n, $n \geq 2$ were constructed. It is proved that all these representations are in some sense equivalent to the Burau representation.

Using the Wada representations of B_n in the automorphism group $\mathrm{Aut}(F_n)$ of a free group the linear representations of B_n are constructed.

There were constructed the local homogeneous representations of virtual braid group VB_n and the ones of welded braid group WB_n. There were also constructed the local representations of singular braid group SB_n.

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Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail address: mikhalchishina@math.nsc.ru

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R_∞ PROPERTY FOR CHEVALLEY GROUPS

TIMUR NASYBULLOV

Let $\varphi : G \to G$ be an arbitrary automorphism of group G. Two elements x and y of group G are called *φ-conjugated* if there exists an element z in G, such that $x = zy\varphi(z^{-1})$. The relation of φ-conjugation is equivalence relation and here we can speak about *φ-conjugay classes.* The number $R(\varphi)$ of φ-conjugacy classes is called the *Reidemeister number* of the automorphism φ. If the Reidemeister number $R(\varphi)$ is infinite for any automorphism φ of group G, then G is said to possess *R_∞ property.*

The question about the groups that possess R_∞ property was formulated by A. Felshtyn and R. Hill [1]. It is known that symplectic group $\mathrm{Sp}_{2n}(\mathbb{Z})$ over the ring of integers possesses R_∞ property [2]. Special and general linear groups $\mathrm{SL}_n(R)$, $\mathrm{GL}_n(R)$ over some integral domains also possess R_∞ property [3].

In this report we consider another class of linear groups – Chevalley groups over the field. And we discuss some sufficient conditions, when such groups possess R_∞ property.

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Novosibirsk State University, Novosibirsk, Russia
E-mail address: timur.nasybullov@mail.ru

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NORMAL AUTOMORPHISMS OF FREE PRODUCTS AND SOME CLASSICAL GROUPS

MIKHAIL NESHCHADIM

An automorphism of an arbitrary group is called normal if all normal subgroups of this group left invariant by it. In this report we discuss the structure of groups of normal automorphisms of free products groups, free nilpotent groups, braid groups and other.

Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail address: neshch@math.nsc.ru

ORLICZ SPACES AND FIRST COHOMOLOGY OF DISCRETE GROUPS

ROMAN PANENKO

Inspired by works of Puls and Martin–Valette (see [1], [2] and [3]) on first L^p-cohomology of discrete groups and p-harmonic functions, we introduce by analogy the notion of the discrete Φ-Laplacian and prove a decomposition theorem for the space of Φ-Dirichlet functions, where Φ is an N-function belonging to the class $\Delta_2(0) \cap \nabla_2(0)$. According to the idea, we study the nonreduced and reduced first cohomology of a (finitely generated) discrete group G with coefficients in the left regular representation of G in the Orlicz space $\ell^\Phi(G)$ and show that if G contains an infinite normal amenable subgroup with infinite centralizer then the cohomology space $H^1(G, \ell^\Phi(G)) = 0$. We also prove a theorem about the triviality of the first cohomology space for a wreath product of two groups the first of which is nonamenable.

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Sobolev Institute of Mathematics, Novosibirsk, Russia
E-mail address: panenkora@gmail.com

ON THE CLASSIFICATION OF HYPERBOLIC ISOMETRIES

SHIV PARSAD

We classify the dynamical action of matrices in $\mathrm{SU}(p,q)$ using the coefficients of their characteristic polynomial. This generalises earlier work of Goldman for $\mathrm{SU}(2,1)$ and the classical result for $\mathrm{SU}(1,1)$, which is conjugate to $\mathrm{SL}(2,\mathbb{R})$. We shall then specialize to the case of $\mathrm{SU}(3,1)$ and will describe the locus of the points where the characteristic polynomials will have repeated roots.

Indian Institute of Science Education and Research, Mohali, India
E-mail address: parsad.shiv@gmail.com

ON UNKNOTTING AND REGION UNKNOTTING NUMBERS OF SOME KNOTS

MADETI PRABHAKAR

In this talk, we provide a new unknotting approach to unknot torus knots and based on this approach, we provide unknotting number of more then 700 knots having crossing numbers from 10 to 16 by showing them in an unknotting sequence of some torus knot. We also discuss the region unknotting number of different classes of 2-bridge knots. In particular, we provide region unknotting number for the classes of 2-bridge knots whose Conway notation is $C(m, n)$, $C(m, 2, m)$, and $C(m, 2, m1)$, respectively.

Indian Institute of Technology Ropar, Punjab, India
E-mail address: prabhakar@iitrpr.ac.in

SYMMETRIC COHOMOLOGY OF GROUPS

MAHENDER SINGH

In this talk, we will discuss a recent construction due to Mihai Staic introducing a symmetric cohomology of abstract groups. We will give a topologized version of this construction giving rise to a symmetric continuous cohomology of topological groups. We will show that the second symmetric continuous cohomology characterise nice topological group extensions and behave nicely for profinite groups.

Indian Institute of Science Education and Research, Mohali, India
E-mail address: mahender@iisermohali.ac.in

ON VASSILIEV INVARIANTS OF BRAID GROUPS OF THE SPHERE

VLADIMIR VERSHININ

We construct a universal Vassiliev invariant for braid groups of the sphere and the mapping class groups of the sphere with n punctures. The case of a sphere is different from the classical braid groups or braids of oriented surfaces of genus strictly greater than zero, since Vassiliev invariants in a group without 2-torsion do not distinguish elements of braid group of a sphere.

This is a joint work with Nizar Kaabi [1].

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Sobolev Institute of Mathematics, Novosibirsk, 630090, Russia
E-mail address: versh@math.nsc.ru

SUBGROUPS OF $\mathrm{PSL}_2(\mathbb{C})$ WHICH ARE EXTREME FOR DISCRETENESS CONDITIONS

ANDREI VESNIN

It is well-known that $\mathrm{PSL}(2,\mathbb{C})$ is isomorphic to the full group of orientation-preserving isometries of the hyperbolic 3-space. Thus, the question about discreteness of a given subgroup is actual in algebraic and geometric contexts both.

In the present talk we will discuss two-generated groups which are extreme for some discreteness conditions. We will mostly interested in groups which uniformize hyperbolic 3-manifolds arising as knot complements and related 3-orbifolds.

For $f, g \in \mathrm{PSL}(2,\mathbb{C})$ let us denote

$$\mathcal{J}(f,g) = |\mathrm{tr}^2(f) - 4| + |\mathrm{tr}[f,g] - 2|.$$

Let $G < \mathrm{PSL}(2,\mathbb{C})$ be a 2-generated non-elementary group. The value

$$\mathcal{J}(G) = \inf_{\langle f,g\rangle = G} \mathcal{J}(f,g),$$

is said to be a *Jørgensen number* of G. Jørgensen numbers originally arise in the following discreteness condition [1]: if non-elementary group G is discrete then $\mathcal{J}(G) \geq 1$. It was shown in [2] that the figure-eight knot complement is the unique hyperbolic 3-manifold whose fundamental group has Jørgensen number equals to one. Jørgensen numbers for some 2-bridge knot groups were calculated in [2].

Let us denote by K the figure-eight knot and by $K(n)$ the hyperbolic 3-orbifold with singular set K and singular angle $2\pi/n$, $n \geq 4$. The knot group has the presentation

$$\pi_1(S^3 \setminus K) = \langle\, f, g \mid [g,f]\, g^{-1} = f\,[g,f]\,\rangle$$

and the orbifold group has the following presentation:

$$\pi^{\mathrm{orb}}(K(n)) = \langle\, f_n, g_n \mid f_n^n = g_n^n = 1,\ [g_n, f_n]\, g_n^{-1} = f_n\,[g_n, f_n]\,\rangle.$$

Both of them have faithful representations in $\mathrm{PSL}(2,\mathbb{C})$.

We will describe behavior of Jørgensen numbers of figure-eight knot orbifold groups.

Theorem 1. [3] *Let $n \geq 4$. Then the following inequalities hold:*

$$1 \leq \mathcal{J}(\pi^{\mathrm{orb}}(K(n))) \leq 4\sin^2(\pi/n) + \sqrt{1 + 4\sin^2(\pi/n)}.$$

Corollary 1. [3] *The following convergence holds:*

$$\lim_{n\to\infty} \mathcal{J}(\pi^{\mathrm{orb}}(K(n))) = \mathcal{J}(\pi_1(S^3 \setminus K)).$$

An analog of a Jørgensen number was introduced in [4] and [5]. For $f, g \in \mathrm{PSL}(2,\mathbb{C})$ such that $\mathrm{tr}[f,g] \neq 1$ denote $\mathcal{G}(f,g) = |\mathrm{tr}^2(f) - 2| + |\mathrm{tr}[f,g] - 1|$. Let $G < \mathrm{PSL}(2,\mathbb{C})$ be a 2-generated group. The value

$$\mathcal{G}(G) = \inf_{\langle f,g\rangle = G} \mathcal{G}(f,g),$$

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is said to be a *GMT number* of G. Gehring and Martin [4] and independently Tan [5] proved that if G is discrete then $\mathcal{G}(G) \geq 1$.

The following results demonstrate behavior of GMT numbers of figure-eight knot orbifold groups.

Theorem 2. [3] *The following equality holds:* $\mathcal{G}(\pi_1(S^3 \setminus K)) = 3$.

Theorem 3. [3] *Let* $n \geq 4$. *Then the following inequalities hold:*

$$1 \leq \mathcal{G}(\pi^{\mathrm{orb}}(K(n))) \leq 3 - 4\sin^2(\pi/n).$$

Corollary 2. [3] *The following equality holds:* $\mathcal{G}(\pi^{\mathrm{orb}}(K(4))) = 1$.

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Sobolev Institute of Mathematics, Novosibirsk, Russia

E-mail address: vesnin@math.nsc.ru

ON THE SUBGROUPS OF THE GROUPS OF BRUNNIAN LINKS

JIE WU

Let $L_n = \{\ell_1, \ell_2, \ldots, \ell_n\}$ be a Brunnian link and $G(L_n)$ be its link group. Suppose that R_i is the normal closure of the meridian of ℓ_i in $G(L_n)$, then R_1, R_2, $\ldots$, R_n are normal subgroups of $G(L_n)$. For each $2 \leq m \leq n$, let $X(L_n)_m$ be the homotopy colimit of the classifying spaces $B(G(L_n))/\Pi$. Here we studied the geometrical property of $X(L_n)_m$ and issued in an algebraic result, i. e., we proved $\cap_{i=1}^{m} R_i = [R_1, \ldots, R_m]_S$, the symmetric commentator subgroup R_1, R_2, $\ldots$, R_n for $1 \leq m \leq n$. This is a joint work with Fengchun Lei and Yu Zhang.

NATIONAL UNIVERSITY OF SINGAPORE, SINGAPORE
E-mail address: matwuj@nus.edu.sg

HOMOTOPY GROUPS, BRAIDS AND LINKS

JIE WU

In this talk, we will talk recent progress on the fundamental connections between homotopy groups, braid and link groups. This talk will be based on the collaborative work of the speaker with his coauthors Valeriy Bardakov, Jon Berrick, Fred Cohen, Fuquan Fang, Fengchun Lei, Fengling Li, Jingyan Li, Roman Mikhailov and Volodia Vershinin.

National University of Singapore, Singapore
E-mail address: matwuj@nus.edu.sg